

# On the Distribution of Estates and the Distribution of Wealth: Evidence from the Dead

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## Abstract

Detailed information about the distribution of estates left at death has commonly served as the basis for the estimation of wealth distributions among the living via the mortality multiplier method. The application of detailed mortality rates by demographics and other determinants of mortality is crucial for obtaining an unbiased representation of the wealth distribution of the living. Yet, in this paper we suggest that a simplified mortality multiplier method, derived using average mortality rates and aggregate tabulations by estate size, may be sufficient to derive compelling estimates of wealth concentration. We show that the application of homogeneous multipliers leads to estimates that are close in level and trend to the concentration of wealth derived in the existing literature with the detailed mortality multiplier method for a variety of countries. The use of mortality rates graduated by estate size does not confute this finding. We also derive the general formal conditions for the similarity between the distributions of wealth of the living and estates at death and discuss the main caveats. These findings may unlock a wide array of aggregate estate tabulations, previously thought to be unusable, for estimating historical trends of wealth concentration.

**Keywords:** Wealth inequality, mortality multiplier method, estates, mortality rates, economic history

**JEL Codes:** D3, H2, N3

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# 1 Introduction

The information from estates (the net value of real and financial property of a deceased person) has commonly served as the basis for the estimation of the distribution of wealth among the living via the mortality multiplier method since the works by Mallet (1908) and Mallet and Strutt (1915) for the United Kingdom. The principle of the method is very basic, even if a number of important conceptual challenges are involved: the set of deceased people are taken as a sample of the living, and each estate is expanded by a multiplier (weight) equal to the inverse probability of death.

Death, however, does not randomly sample the population. Older individuals, as well as males and people from poorer backgrounds, have, other things being equal, higher mortality risk. Differential mortality multipliers should be used to transform the estate data into estimates of wealth-holding. When demographic multipliers (i.e., related to age and gender) were first employed in the United Kingdom, it was seen as overcoming a “fatal” objection to the use of estate data with a constant multiplier. In fact, death occurrence can be seen as an effective sampling of the living population only under the assumption that death is random within specific cells of observed demographic strata. For instance, recent work by Saez and Zucman (2016) brought to the fore long-standing concerns about the mortality multiplier method, suggesting that the failure to appropriately control for decreasing mortality of wealthy individuals may severely underestimate the top wealth shares (see also Atkinson and Harrison (1978)).

These considerations point to the fact that the distribution of wealth of the living is conceptually different from that of decedents. In other words, the application of detailed multipliers (that is, those taking into account the dimensions that affect the probability of dying) can increase or decrease the estimates of wealth concentration or inequality, as well as affect their trends over time. Yet, the distribution of estates at death has recently come under extensive scrutiny. Recent research has surprisingly highlighted that the application of mortality multipliers may not significantly alter the distribution of estates as previously thought (Alvaredo, Atkinson and Morelli, 2018). In this paper, we scrutinize the latter result and implement a *simplified multiplier method* which makes use of the average mortality rate in the population. We first provide empirical evidence that the levels and trends of wealth concentration derived with a homogeneous multiplier are sufficiently close to those obtained where the detailed mortality multiplier method was applied in existing works. This exercise was carried out for Australia, France, Italy, Korea, the United Kingdom, and the United States.

The paper then addresses more formally the following issues: What is the nature of the relationship between the distribution of estates at death and that of the wealth of the living through the mortality multiplier method? What are the general conditions under which the concentration of estates at death provides the same informative content as the concentration of the wealth of the living? What drives the relevance and the direction of the potential bias created by the lack of appropriate control for the growing longevity of wealthy individuals?

The formalization of the problem leads to several insights. First, we show that the top wealth shares are trivially equal to the top estate shares in the special case when homogeneous multipliers are applied to the population of decedents. Thus, for simplicity, we may refer interchangeably to the distribution of estates and the distribution of wealth derived with homogeneous multipliers. Second, we derive the formal conditions for the equality between top wealth and estate shares and then test the relevance of these conditions using empirical data. In fact, when heterogeneous multipliers are used (e.g., when mortality rates differ by demographic characteristics), the concentration of wealth and estates may not be identical.

We begin by considering the case of heterogeneous mortality multipliers differentiated by demographic characteristics. This is an important starting point as mortality rates by age and gender generally map most of the variability in mortality observed in a country in a given year. Moreover, these data are available throughout history for many countries. We then consider the case of heterogeneous multipliers differentiated by socio-economic characteristics. Accounting for this heterogeneity would create a more accurate picture of mortality multipliers and hence lead to a more realistic estimation of top wealth shares. We find that the difference between the top wealth shares obtained with heterogeneous or homogeneous multipliers cannot be large under realistic assumptions and given the observed regularities of the interrelation between the wealth distribution and demographic characteristics.

After controlling for realistic mortality-wealth gradients, we find that the newly estimated top wealth share is close to the one derived with the use of the average multiplier among adults. Although individuals at the top of the estate distribution have higher mortality rates, as they are relatively older on average, this is counterbalanced by their higher economic status, which may lead to healthier lives and better medical care, reducing their probability to die, other things being equal. As a result, the differences between the mortality multipliers at the top of the estate distribution and the average mortality multiplier of the entire decedent population are small enough to create only a limited discrepancy between the two top wealth shares estimated with differential and average multipliers.

We believe these findings may unlock a wide array of aggregate tabulations that were previously thought to be unreliable and unusable. Information about the distribution of wealth is scarce, for the recent period and even more so for historical series. Yet, many countries have published detailed data on the distribution of estate taxes. These are only rarely accompanied by demographic characteristics such as age and gender. Thus, one cannot apply heterogeneous mortality rates to the estate tax data. The simplified multiplier method may be implemented in such cases for estimating historical trends of wealth concentration.

Nonetheless, one important caveat remains. Although, as discussed, changes in the multipliers do not dramatically affect the shape of the wealth distribution, they may have a large impact on estimated aggregate variables, such as wealth totals and the ratio between the average estate and the average wealth among the living. These aggregate variables are usually of lesser interest

if the goal is obtaining information on the distribution of wealth. However, the derivation of macroeconomic aggregate series are of direct interest to economists (see [Piketty and Zucman \(2014\)](#) and [Alvaredo, Garbinti and Piketty \(2017\)](#)), and, we argue, may well serve as an indirect test of the appropriateness of the multipliers used to derive distributional measures. Mortality multipliers thus matter in various respects.

## 2 The mortality multiplier method

The mortality multiplier method makes use of the information on the wealth and the demographic characteristics of decedents reported to the tax authorities for the administration of inheritance or estate taxes. The decedent population can then be re-weighted and become representative of the living population to estimate its distribution of wealth. Let's consider the population of  $N_E$  decedents and the total value of their estates,  $W_E$ :

$$W_E = \sum_{i=1}^{N_E} w_{E,i}. \quad (2.1)$$

The estates  $w_{E,i}$  are arranged in descending order, i.e.,  $w_{E,i} \geq w_{E,j}$ , if  $i < j$ . The following relationship holds:

$$W = \sum_{i=1}^{N_E} m_i w_{E,i}, \quad (2.2)$$

where  $W$  is the total wealth among the living population, and  $m_i \equiv \frac{1}{p_i}$  is the mortality multiplier of individual  $i$ , equal to the inverse of the individual mortality rate.

The mortality rates vary across a set of socio-demographic characteristics. Therefore each multiplier represents the number of living individuals who share the same socio-demographic characteristics of decedent  $i$ .

The average mortality multiplier,  $\bar{m}$ , is defined as the ratio between the number of the living,  $N$ , and the number of the deceased,  $N_E$ . Similarly, we define its inverse, the overall mortality rate,  $\bar{p}$ , as  $N_E/N$ .<sup>1</sup>

We are interested in estimating the wealth share of the top quantile  $0 < q < 1$ , where  $q = 0.1$  corresponds to 10%,  $q = 0.01$  corresponds to 1%, etc. It is natural to think that the value of multipliers will affect the number of decedents that will be needed in order to account for the top  $q$  quantile among the living (i.e., for  $qN$  living individuals). For example, if the multipliers of the rich decedents are high, compared to the average multipliers in the population, fewer decedents would be required to account for the top  $q$  quantile among the living than when the multipliers of

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<sup>1</sup>  $\bar{m}$ , the average mortality multiplier, is equal to the arithmetic mean of individual multipliers:  $\bar{m} = \frac{1}{N_E} \sum_{i=1}^{N_E} m_i = \frac{N}{N_E} = \frac{1}{\bar{p}}$ .  $\bar{p}$ , the overall mortality rate, is the arithmetic mean of mortality rates among the living ( $\frac{1}{N} \sum_{j=1}^N p_j$ ).

the rich decedents are lower. This number is represented by the index  $I_q$  such that<sup>2</sup>

$$\sum_{i=1}^{I_q} m_i = qN. \quad (2.3)$$

This way we can define the top  $q$  wealth share as

$$(1 - L_q)^W = \frac{\sum_{i=1}^{I_q} m_i w_{E,i}}{W}. \quad (2.4)$$

### 3 The concentration of wealth in a simplified mortality multiplier method: heterogeneous vs. average multipliers

The application of the full mortality multiplier method is conditioned on the availability of detailed mortality data as well as detailed estate data by demographic characteristics. However, such information may not be readily available for estate tabulations. It may not be differentiated by demographic characteristics, or detailed mortality data may not exist for a particular country or year.

Work by [Alvaredo, Atkinson and Morelli \(2018\)](#) has shown that the concentration of estates at death and the derived concentration of wealth at the top following the application of mortality multipliers (based on gender, age, social class differentials, or wealth differentials) are very close to one another. In their words, “the application of mortality multipliers does not alter the picture concerning the distribution of the wealth of the living, as commonly believed.” As described by [Cowell \(1978\)](#), referring to [Atkinson and Harrison \(1978\)](#), “though the particular refinement of mortality multiplier that is used considerably affects the calculation of total wealth, the resultant effect on top wealth shares is not all that great.” An implicit recognition of this similarity can be also found in [Piketty, Postel-Vinay and Rosenthal \(2006\)](#) and [Moriguchi and Saez \(2008\)](#), who treat the distribution of estates, estimated using estate tax records, as a de facto equivalent to the distribution of wealth.

Thus, an alternative and simplified solution when detailed mortality data are unavailable would be to rely on average mortality rates, assuming  $m_i = \bar{m}$ . Under this working assumption, the top  $q$  quantile among decedents represents the top  $q$  quantile (i.e.,  $I_q = qN_E$ ), and the top wealth shares will take the following form

$$(1 - L_q)_{avg}^W = \bar{m} \frac{\sum_{i=1}^{qN_E} w_{E,i}}{W}. \quad (3.1)$$

We compare top wealth share series for several countries for which the full mortality multiplier method was performed in the existing literature, using heterogeneous multipliers, to series derived

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<sup>2</sup>If there is no equality,  $I_q$  is defined as the smallest index such that  $\sum_{i=1}^{I_q} m_i > qN$ .

using the simplified mortality multiplier method, i.e., using average mortality rates.<sup>3</sup> Importantly, noting that  $(1 - L_q)_{avg}^W$  is formally equivalent to the top  $q$  estate share, this exercise would be equivalent to comparing wealth and estate top shares as done in [Alvaredo, Atkinson and Morelli \(2018\)](#).<sup>4</sup>

Figure 1 presents these results for Australia, France, Italy, the Republic of Korea (South Korea), the United Kingdom, and the United States. For each country, the evolution of top wealth shares reported in the literature is compared to estimates of top wealth shares derived in this paper using the average mortality multiplier. The empirical exercise highlights that in all countries the top wealth shares estimated with the simplified mortality multiplier method strongly co-move with those reported in the literature, and they are generally similar in level. The largest differences appear in the cases of the United States and South Korea.

The quality and features of estate data in each of these countries differ substantially (see Appendix A). For example, in Italy the data cover roughly 60% of decedents every year; however, only tabulations are publicly available. In France the data cover a much smaller share (about 10%) of the decedent population, yet micro data are available. In the United States, only a tiny share of the decedent population is covered (roughly 0.2% in recent years) and public aggregate tabulations as well as detailed micro data are available to researchers. Despite the specific differences between the data sources, the application of the mortality multiplier method is similar and requires the same information—the values of estates at the top of the estate distribution (or equivalent tabulations), the corresponding mortality multipliers at the top of the estate distribution, and the total personal wealth.

Different works make use of different adjustments to the data to allow for underreporting, tax avoidance and evasion. In the United States, [Kopczuk and Saez \(2004\)](#) include estimates of wealth held in trusts and the cash surrender value of pensions and life insurance assets. In France, [Garbinti, Goupille-Lebret and Piketty \(2020\)](#) impute missing net wealth to provide consistency with official national balance sheet data for the household sector. Other works, such as [Acciari, Alvaredo and Morelli \(2020\)](#) for Italy, provide a full array of adjusted, unadjusted, and imputed series. As we are able to make use of the unadjusted series which is derived from the pure application of mortality multipliers here, we primarily use the case of Italy for our empirical analysis in the following sections.

To gain a better understanding of how sensitive the results can be to the choice of multipliers, we use equations Eq. (2.4) and Eq. (3.1) and derive the conditions for the equality of the top wealth

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<sup>3</sup>See Appendix B for a discussion of the different possible choices of average multiplier and the effect this has on estimates of wealth concentration. We continue our analysis using  $\bar{m} = N/N_E$  as explained above, where  $N$  will be taken as the living adult population and  $N_E$  as the number of adult decedents.

<sup>4</sup>When  $m_i = \bar{m}$ , then  $W = \sum_{i=1}^{N_E} m_i w_{E,i} = \bar{m} \sum_{i=1}^{N_E} w_{E,i} = \bar{m} W_E$ . We obtain

$$(1 - L_q)_{avg}^W = \frac{\sum_{i=1}^{I_q} m_i w_{E,i}}{W} = \frac{\bar{m}}{\bar{m}} \frac{\sum_{i=1}^{q N_E} w_{E,i}}{W_E} = (1 - L_q)^E. \quad (3.2)$$

See also Appendix C for more details.

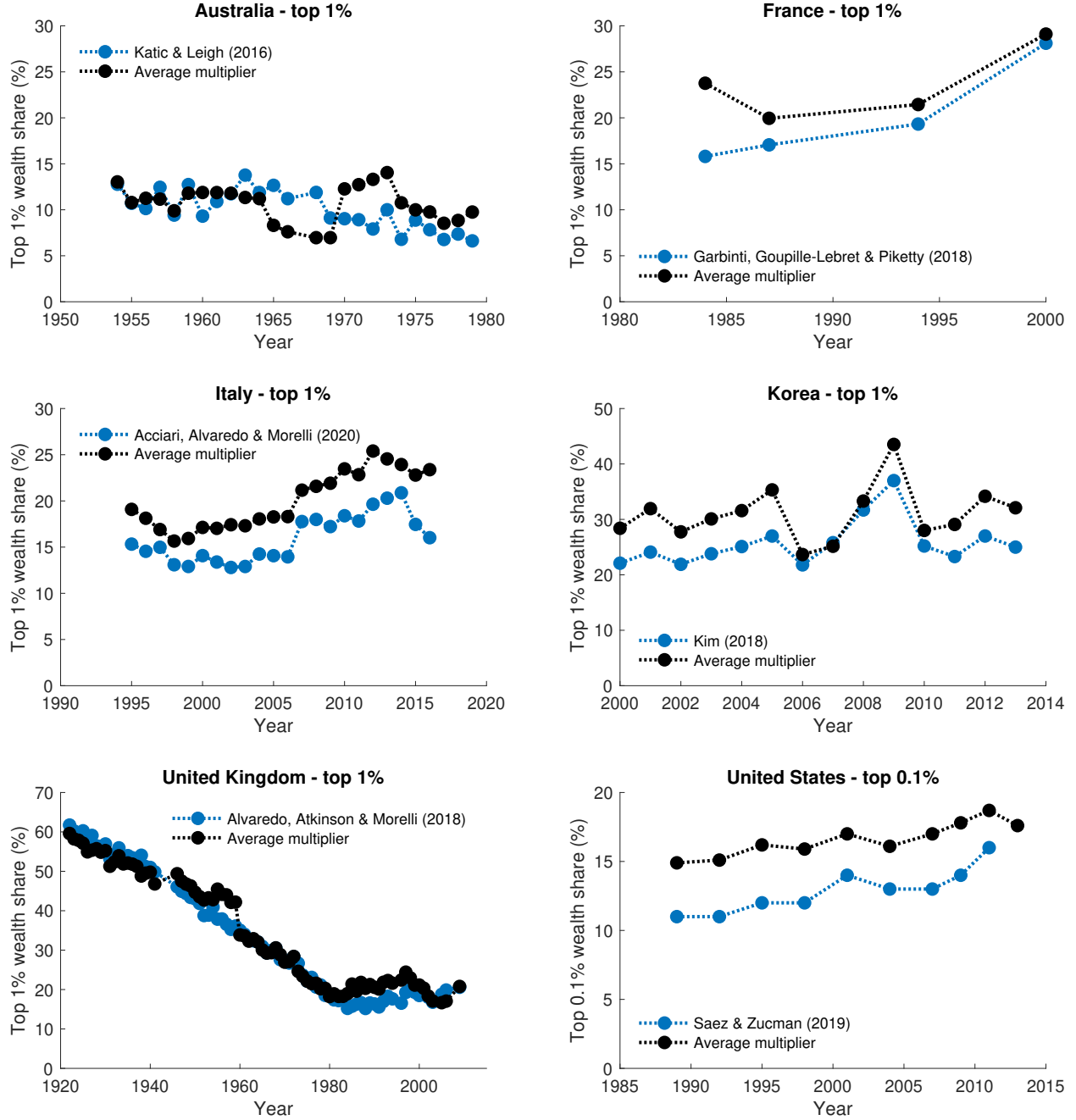


Figure 1: The top wealth shares in Australia, France, Italy, South Korea, the United Kingdom, and the United States. Tabulations and the top wealth shares were taken from [Katic and Leigh \(2016\)](#), [Garbinti, Goupille-Lebret and Piketty \(2020\)](#), [Acciari, Alvaredo and Morelli \(2020\)](#), [Kim \(2018\)](#), [Alvaredo, Atkinson and Morelli \(2018\)](#), and [Saez and Zucman \(2019\)](#), respectively. The estimated top wealth shares were produced using the mortality multiplier method assuming the average multiplier for all observed decedents. The mortality data were taken from [The Human Mortality Database \(2018\)](#).

shares with the average multiplier and with heterogeneous multipliers:

$$\frac{\sum_{i=1}^{I_q} m_i w_{E,i}}{W} = \bar{m} \frac{\sum_{i=1}^{qN_E} w_{E,i}}{W} \iff \sum_{i=1}^{I_q} \frac{m_i}{\bar{m}} w_{E,i} = \sum_{i=1}^{qN_E} w_{E,i}^*, \quad (3.3)$$

and the equality is trivially satisfied if multipliers do not vary across the population (i.e.,  $m_i = \bar{m}$ ).

We define

$$\bar{w}_{qN_E} = \frac{\sum_{i=1}^{qN_E} w_{E,i}}{qN_E} ; \quad \bar{w}_{I_q} = \frac{\sum_{i=1}^{I_q} w_{E,i}}{I_q} \quad (3.4)$$

and

$$\bar{w}_{qN_E} - \bar{w}_{I_q} = \frac{I_q}{\bar{m}qN_E} \text{Cov} [m_i, w_{E,i}] , \quad (3.5)$$

where  $\text{Cov} [m_i, w_{E,i}] = \frac{1}{I_q} \sum_{i=1}^{I_q} \left( m_i - \frac{1}{I_q} \sum_{j=1}^{I_q} m_j \right) (w_{E,i} - \bar{w}_{I_q})$ .

Now, rearranging terms, it is possible to explicitly denote the difference between the top wealth shares and obtain via the same notation and using the same expansion:

$$\begin{aligned} (1 - L_q)^W - (1 - L_q)_{avg}^W &= \frac{\bar{m}qN_E}{W} (\bar{w}_{I_q} - \bar{w}_{qN_E}) + \frac{I_q}{W} \text{Cov} [m_i, w_{E,i}] \\ &= \frac{I_q}{W} [\bar{m}_{I_q} (\bar{w}_{I_q} - \bar{w}_{qN_E}) + \text{Cov} [m_i, w_{E,i}]] , \end{aligned} \quad (3.6)$$

where  $\bar{m}_{I_q}$  is the average multiplier at the top of the estate distribution ( $\sum_{i=1}^{I_q} m_i / I_q$ ).

The right-hand side of Eq. (3.6) shows that the difference between top wealth shares depends on an average level effect of the multipliers,  $\bar{m}_{I_q} (\bar{w}_{I_q} - \bar{w}_{qN_E})$ , and on the covariance,  $\text{Cov} [m_i, w_{E,i}]$ . The average level effect is such that the closer the average of the multipliers at the top is to the average multiplier, the closer the index  $I_q$  is to  $qN_E$ , and hence, the closer the difference  $\bar{w}_{I_q} - \bar{w}_{qN_E}$  would be to zero, leading to a smaller difference between the two top wealth shares estimated with heterogeneous and homogeneous multipliers.

In practice, the average multiplier at the top tends to be lower than  $\bar{m}$ . This is a straightforward result of life cycle effects—mortality is predominantly determined by age, and older people tend to be richer, on average (Shorrocks, 1975; Modigliani, 1986). Therefore, the top of the estate distribution is likely to be composed of people that are older than the average age among the adult population. In order to account for the top  $qN$  living individuals, we would then need more than  $qN_E$  decedents (note that  $\bar{m} = \frac{N}{N_E}$ ). For this reason, the difference  $\bar{w}_{I_q} - \bar{w}_{qN_E}$  would tend to be negative, and using the average multiplier would lead to overestimation of the top wealth shares when compared to heterogeneous demographic multipliers.

The covariance ( $\text{Cov} [m_i, w_{E,i}]$ ) also tends to be negative in practice, but it is generally small. Mortality rates increase exponentially with age above the age of 40 (see Appendix D). Wealth increases with age more weakly and the variability of age within wealth groups is large. Thus, the covariance between estates and multipliers at the top of the estate distribution is negative but close to zero, which may lead to a similarity between the top wealth shares derived above. Figure 2 illustrates this point for France in different years, using a sample of the richest decedents obtained from the estate tax records. It shows the large variability in age within top wealth groups, and the weak dependence of age on wealth rank at the top of the estate distribution.



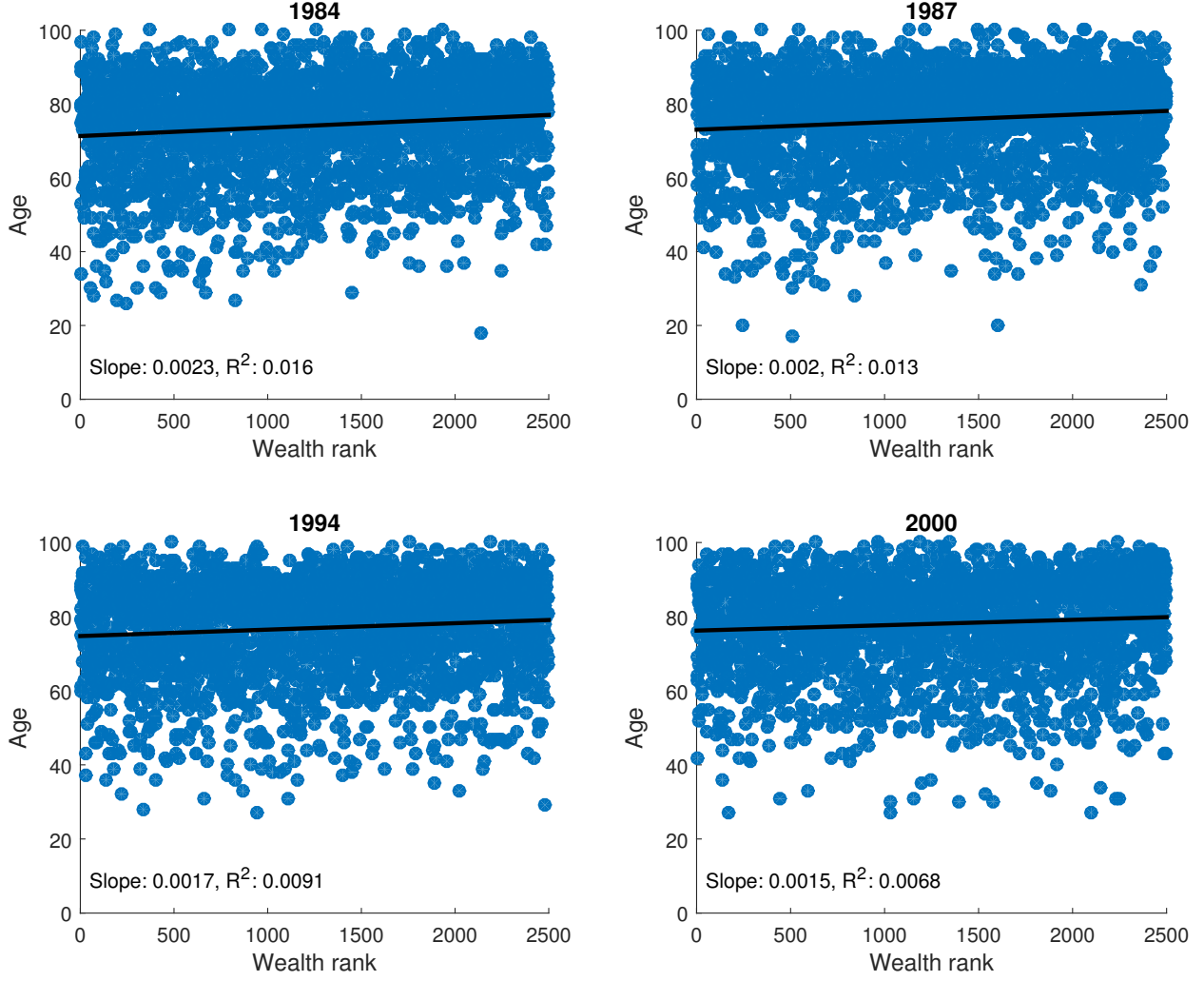


Figure 2: The age of a sample of the richest decedents in France according to their wealth rank (1 – least wealthy). The slope and  $R^2$  are given for a linear fit of the data in each year.

We note that a similar derivation of the comparison between top wealth shares (Eq. (3.6)) can be used to compare the coefficient of variation (CV) of the wealth distribution with homogeneous and heterogeneous multipliers (see Appendix E). It clarifies the intuition for the result obtained for top shares above. In particular, it shows that the difference between the CV of wealth and estates is mainly driven by the multipliers at the top of the estate distribution. This supports the observation that a similarity between the multiplier at the top of the estate distribution and the average mortality multiplier would result in a similarity between the estimated concentration of wealth and the concentration of estates.

### 3.1 Accounting for multipliers graduated by wealth levels

Mortality rates are clearly influenced by demographic factors, such as gender and age. However, social and economic conditions can also exert a substantial influence on the longevity of individuals (Chetty et al., 2016). In particular, higher wealth levels may be systematically associated with lower mortality rates, over and above the effect of demographics and other factors. Failure to account for this additional source of heterogeneity in mortality rates may lead to systematic biases in the application of the mortality multiplier method (Atkinson and Harrison, 1978; Saez and Zucman, 2016, 2019). To account for the contribution of wealth to lower mortality over and above the effect of age, we use Italian estate tabulations from Acciari, Alvaredo and Morelli (2020) and apply mortality rate adjustment factors for wealth used by Garbinti, Goupille-Lebret and Piketty (2020).

The formalization described by equation Eq. (3.6) is well suited to take this issue into account and explain the main findings. Accounting for the mortality-wealth gradient does, indeed, increase the covariance, other things being equal, possibly creating a positive association between estate values and mortality multipliers at the top of the estate distribution. At the same time, the gradient increases the average multiplier at the top, bringing it closer to the average mortality multiplier,  $\bar{m}$ . This would, in turn, increase the difference  $\bar{w}_{I_q} - \bar{w}_{qN_E}$ , i.e., make it closer to zero. We should also expect, therefore, that the top wealth shares derived via wealth-gradient multipliers will be higher than those derived through demographic multipliers alone.

The results are presented in Figure 3, where the derived series of top wealth shares using a mortality-wealth gradient is compared to those derived with the average multiplier as well as heterogeneous multipliers by demographic characteristics. The results show that a steep mortality-wealth gradient can have a salient effect on the top wealth shares.<sup>5</sup>

Nevertheless, the wealth effect on mortality can counterbalance the small negative correlation between multipliers and estates at the top. Combined, the wealth and age effects on mortality may lead to correlation that is very close to 0. If, indeed, the decreasing mortality of wealthy individuals is not accounted for, the correlation would be underestimated. At the same time, decreasing mortality by wealth acts to increase the life expectancy of older, wealthy individuals. This, in turn, leads to the decrease of the covariance between multipliers and estates at the top. For these reasons, a large positive covariance between estates and multipliers at the top, which will lead to large positive differences between the top wealth shares with and without heterogeneous multipliers, is implausible.

More surprisingly, Figure 3 shows that the top wealth series derived using the simplified mortality multiplier method using average multipliers provides very similar results to those obtained by applying detailed multipliers by demographic and wealth status. The wealth gradient of mortality rates reduces the mortality rates of the richest individuals, increasing multipliers. This means that wealth provides an “age premium” to older rich individuals. In turn, this leads to mortality

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<sup>5</sup>We note that it is possible that the mortality-wealth gradient described in Garbinti, Goupille-Lebret and Piketty (2020) may not be representative of Italy.

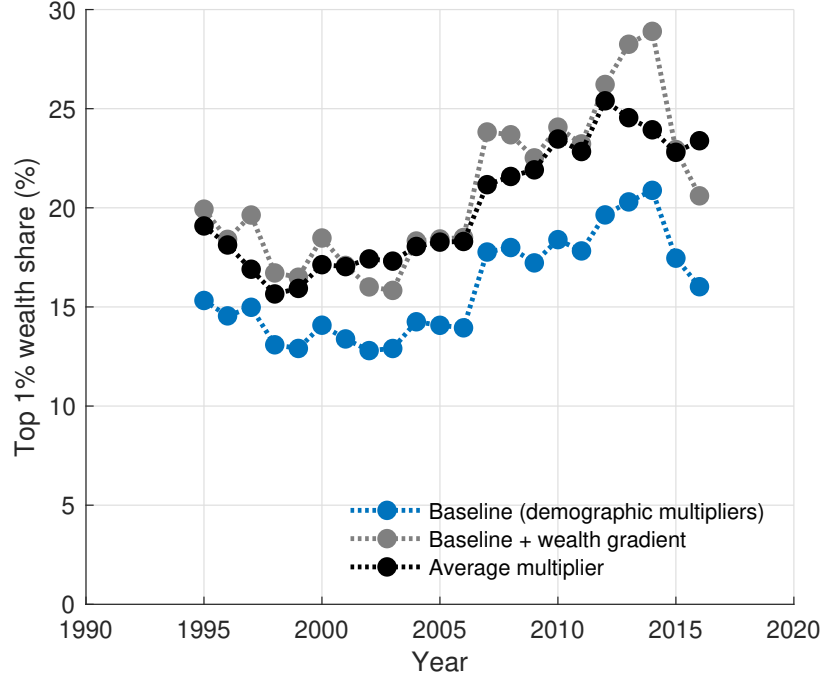


Figure 3: The top 1% wealth shares in Italy estimated using different multiplier choices: heterogeneous demographic multipliers (blue); wealth-adjusted heterogeneous demographic multipliers (gray); average (homogeneous) multiplier (black).

multipliers at the top of the estate distribution that are close to the average multiplier in the overall adult population.

## 4 Additional effects of using average multiplier and graduated multipliers

The mortality multiplier method, and, in particular, the application of a mortality-wealth gradient to the data, may give rise to several problems, which are not commonly taken into account. The mortality-wealth gradient leads to mortality multipliers that are higher than without it among the wealthiest individuals. This makes the resulting average wealth among the living higher than without applying the gradient. If the gradient applied is too steep,  $\mu$ , the ratio between the average wealth at death to the average wealth among the living, will be less than 1.  $\mu < 1$  is a very unlikely case, implying that the decedents are poorer, on average, than the living. This is possible, in theory, if the rich are very unlikely to die, but that is an extreme case, undocumented so far (see, for example, [Alvaredo, Garbinti and Piketty \(2017\)](#); [Alvaredo, Atkinson and Morelli \(2018\)](#)). An additional potential problem is an overestimation of identified wealth. This may be an issue if the coverage of the data is relatively high. If the coverage is particularly low, e.g., in France or in the United States, the problem may be less visible but might still exist and go unnoticed.

Figure 4 demonstrates these issues for Italy. It shows the evolution of various variables under the mortality multiplier method with different multiplier choices. In particular, it shows that the mortality-wealth gradient used might be too steep in the Italian case, as it implies that  $\mu$  is less than 1 for almost the entire period. In addition, when including the mortality-wealth gradient (as well as when using the average multiplier) it is possible for the identified wealth to be higher than the total personal wealth from the national accounts. This is possible if the unobserved population has negative net wealth. Yet, this is also a rather extreme case, which requires verifying the validity of the mortality-wealth gradient applied. In particular, this may serve as a warning sign.

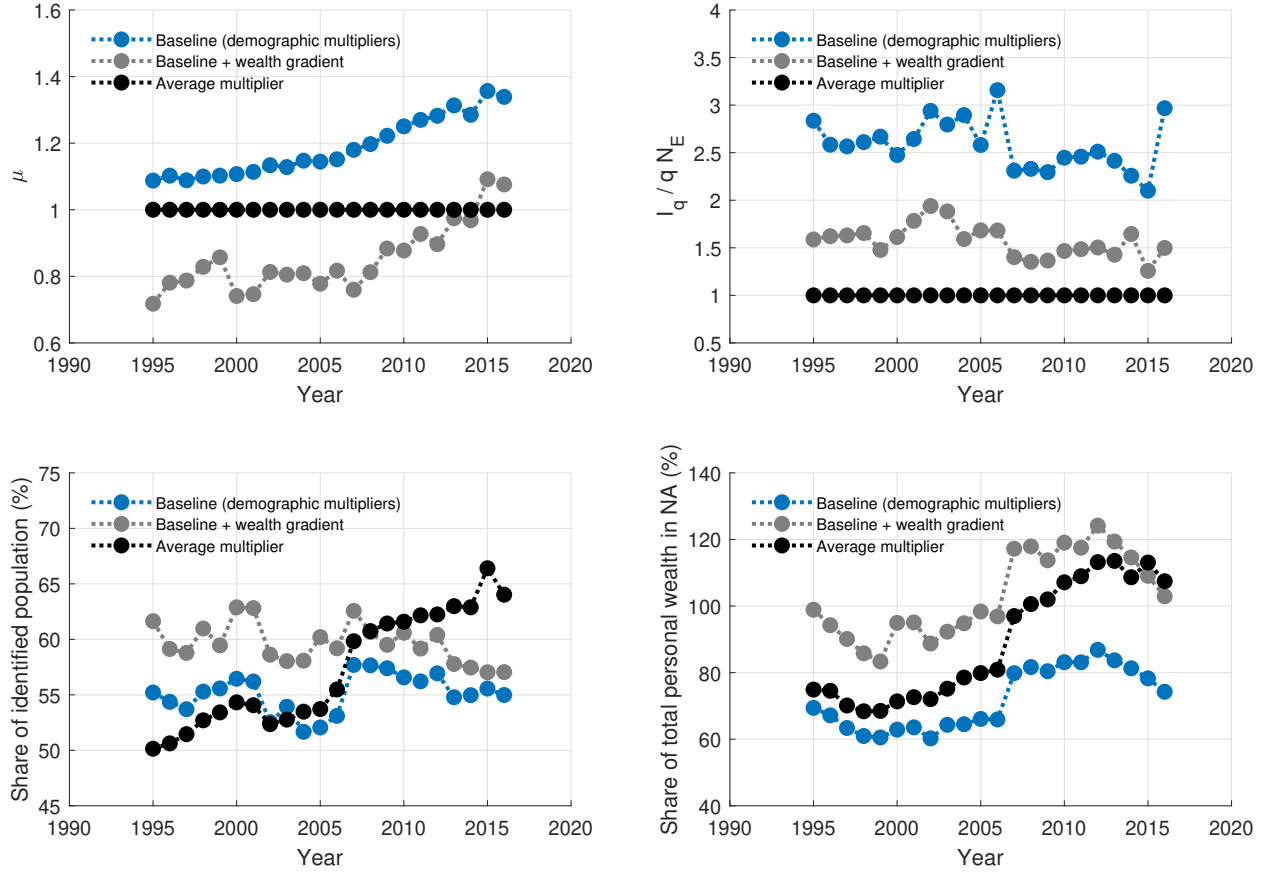


Figure 4: The evolution of various variables in Italy using different multiplier choices. Top left:  $\mu$ ; Top right:  $\frac{I_q}{q N_E}$ ; Bottom left: The share of identified population from total population; Bottom right: The identified wealth as share of total personal wealth from the national accounts. Mortality data are taken from [The Human Mortality Database \(2018\)](#), the estate tabulations and demographic data as well as the total personal wealth are taken from [Acciari, Alvaredo and Morelli \(2020\)](#). The mortality-wealth gradients used were those used for France in [Garbinti, Goupille-Lebret and Piketty \(2020\)](#).

## 5 Conclusion

By clarifying the functioning of the mortality multiplier method and its structural limitations, this paper contributes to the evolving literature on wealth distribution estimation as well as on the important ongoing methodological debate surrounding the mortality multiplier method itself.

On the one hand, the validation of the empirical finding that top estate and wealth shares co-move and have similar levels (Alvaredo, Atkinson and Morelli, 2018) can be crucial for the expansion of severely sparse data series on wealth distribution, both across countries and over time. Indeed, in the case of the United Kingdom, the close relationship between estate distribution and wealth distribution provides a strong measurement benchmark in order to extend the wealth concentration series back in time to 1895 and to fill in missing years. Similarly, construction of long series can become possible in other countries when the relevant information for the application of the mortality multiplier (i.e., detailed estate tabulations or detailed mortality rates) method cannot be retrieved.

On the other hand, the answers to the main questions raised in this paper are crucial for the reliability of the mortality multiplier method, which remains one of the few viable benchmark methods to estimate wealth concentration, particularly in a historical perspective. This is important, as the use of different methodologies and sources of data for the estimation of wealth distribution remains essential for illuminating the limitations of each data source and methodology and to inform us about the levels and trends of wealth concentration. Moreover, and as a matter of fact, the mortality multiplier method is often the only one available to yield estimates of wealth distribution and concentration for specific countries or time periods.

We specifically discuss the relevance of unobserved heterogeneity in mortality rates, such as the potential wealth effect on mortality that is operating over and above the effect of demographic characteristics. Accounting for a mortality-wealth gradient would create a more accurate picture of mortality multipliers and hence lead to a more realistic estimation of top wealth shares. We find that the difference between the top wealth shares obtained with or without mortality-wealth gradients cannot be large under realistic assumptions and given the observed regularities of the interrelation between the wealth distribution and demographic characteristics. While the mortality-wealth gradient can be steep for younger age groups, it is not as steep for older age groups, as economic status does not counterbalance the biological limitations to human longevity. Therefore, adjusting the multipliers at the top of the distribution and taking into account the mortality-wealth gradient is muted by the fact that relatively older people are more represented among the richest decedents. Also, within the top of the estate distribution, there is only a weak dependence of age on wealth rank. As a result, the multipliers at the top may continue to be poorly correlated with wealth ranks, and may continue to be close to the average multiplier of the overall population.

This leads to the important finding that taking into account both demographic multipliers and mortality-wealth gradient yields very similar top wealth shares to those obtained using the average multiplier. Although individuals at the top of the estate distribution have higher mortality rates

(as they are relatively older on average), this is counterbalanced by their higher economic status, which may lead to healthier lives and better medical care, reducing their probability to die, other things being equal. As a result, the differences between the mortality multipliers at the top of the estate distribution to the average mortality multiplier of the entire decedent population are small enough to create only a limited discrepancy between the two top wealth shares estimated with refined multipliers and the average multiplier. These results are of particular relevance for the estimation of historical series of wealth concentration. They would enable the use of a wide array of aggregate estate tabulations that were previously thought to be unreliable and unusable.

We end with an important practical remark. Information about the wealth gradient of mortality rates is scarce, and we know little about how this gradient has evolved over time. In only a few cases, such as France and the United States during the last several decades, do we have some information about the income gradient of mortality and its trend. Hence, in practice, the application of a mortality-wealth gradient is surrounded with considerable uncertainty. Thus, applying such gradients may not necessarily be satisfactory. At the same time, applying an average multiplier to the entire decedent population, as we suggest, can also create a similar problem. For these reasons we highlight the need to be careful and transparent when using the mortality multiplier method and to make use of as much data as possible for consistency. Applying the population average multiplier to all decedents may indeed provide reliable estimates of top wealth shares, especially in a historical context. Yet, they still need to be taken with the necessary caution. We also note that the discussion in this paper presupposes that the information provided by the value of estates at death is valid. In some cases it could be argued that the estates recorded by the tax administration are particularly imperfect, due to high level of exemptions, evasion, or through the effects of tax planning. In such cases the concerns about the effect of mortality multipliers become less crucial compared to the inaccuracy of observed estates in describing the personal wealth of decedents. However, these discussions exceed the scope of this paper.

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## A Estate data coverage

Estate data usually represent only a share of the decedent population, with substantial heterogeneity across countries. Figure 5 shows the share of the decedent population represented in the data in the group of countries analyzed in Figure 1.

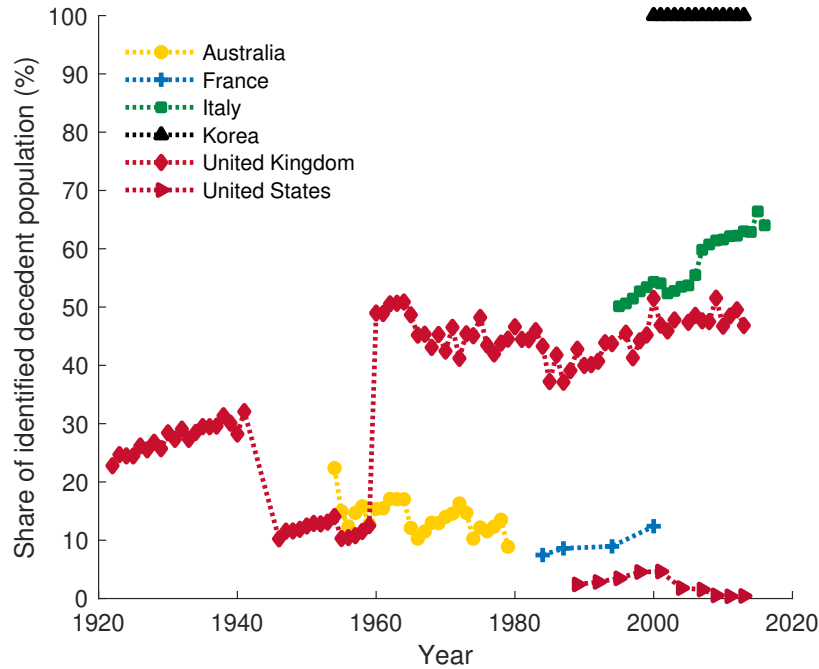


Figure 5: The share of decedents covered in the estate data in Australia, France, Italy, Korea, the United Kingdom, and the United States. The data were taken from [Katic and Leigh \(2016\)](#), [Garbinti, Goupille-Lebret and Piketty \(2020\)](#), [Acciari, Alvaredo and Morelli \(2020\)](#), [Kim \(2018\)](#), [Alvaredo, Atkinson and Morelli \(2018\)](#), and [Saez and Zucman \(2016\)](#), respectively, combined with mortality data from [The Human Mortality Database \(2018\)](#).

## B Which homogeneous multiplier?

For implementing the simplified mortality multiplier method one has to choose the homogeneous multiplier to be applied. The choice in the average multiplier of the adult population,  $\bar{m}$ , is only one possible choice. In the absence of detailed demographic data for the decedents included in the estate tax records, it is possible to use an approximation for their mortality multiplier to estimate the top wealth shares. Even when demographic data are available, homogeneous multipliers can simplify the estimation process. We list below several possible choices of a homogeneous multiplier and present the differences between them, and demonstrate their corresponding resulting top wealth shares in the case of Italy:

- $m_1$ : A simple and natural choice of such a multiplier is the average multiplier  $\bar{m}$ , which is the ratio between the population size of the living and the dead. Considering such a multiplier makes an implicit assumption that the mortality rate of the observed decedents is similar to that of the unobserved decedents.
- $m_2$ : If detailed demographic data are available, it is possible to take the arithmetic average of the individual multiplier  $m_i$ .  $m_2$  is expected to be lower than  $m_1$ , since the average multiplier among the observed decedents tends to be lower than the average multiplier; however, this is not always the case.
- $m_3$ :  $m_2$  changes the identified wealth compared to the case in which the individual multiplier  $m_i$  is considered, because  $\sum_i^{NE} m_i w_{E,i} \neq \frac{\sum_i^{NE} m_i}{N_E} \sum_i^{NE} w_{E,i}$ . Another possible choice of homogeneous multiplier would be a multiplier that is consistent with the identified wealth: 
$$m_3 = \frac{\sum_i^{NE} m_i w_{E,i}}{\sum_i^{NE} w_{E,i}}.$$
- $m_4$ : If no demographic data are available, but mortality data are, it is possible to assume that the representative multiplier of the observed decedents is the multiplier that corresponds to an individual whose age is the average age at death, based on the mortality data. Typically, since this age is higher than the average age of decedents in the tax records, this multiplier will be substantially lower than the other choices of multiplier.
- $m_5$ : The same  $m_2$  but after adding a mortality-wealth gradient to the demographic data to obtain heterogeneous individual multipliers.

The evolution of these multipliers over time and the resulting top 1% wealth shares are presented in Figure 6. The choice of a homogeneous multiplier matters for the estimated top shares. Yet almost all the options considered lead to levels of inequality that closely follow the results when heterogeneous multipliers are used.

The major exception is  $m_4$ , the multiplier that corresponds to the average age of decedents in a given year.  $m_4$  is much lower than the other suggested choices, as it effectively ignores the

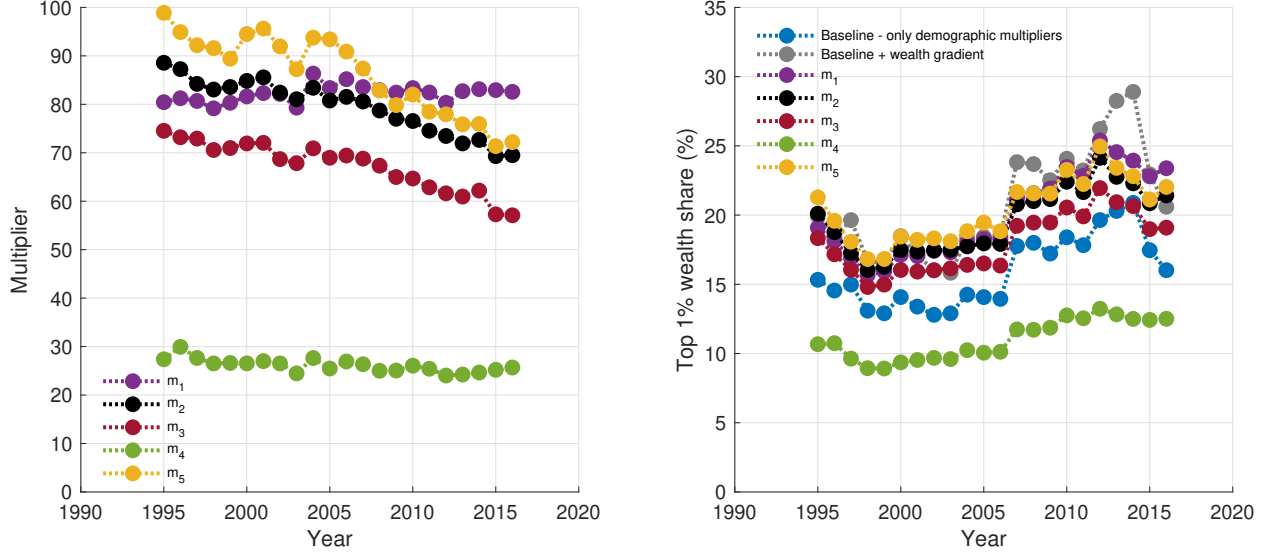


Figure 6: Homogeneous mortality multipliers in Italy 1995–2016. Mortality data were taken from [The Human Mortality Database \(2018\)](#), and the estate tabulations and demographic data were taken from [Acciari, Alvaredo and Morelli \(2020\)](#). The mortality-wealth gradients used were those used for France in [Garbinti, Goupille-Lebret and Piketty \(2020\)](#). The homogeneous multipliers used are:  $m_1$ —the average multiplier  $\bar{m}$ , the ratio between the population size of the living and the dead;  $m_2$ —the arithmetic average of the individual multiplier  $m_i$ ;  $m_3 = \frac{\sum_i^{NE} m_i w_{E,i}}{\sum_i^{NE} w_{E,i}}$ ;  $m_4$ —the multiplier that corresponds to an individual whose age is the average age at death, based on the mortality data;  $m_5$  – the same  $m_2$  but after adding a mortality-wealth gradient to the demographic data.

presence of younger decedents among the wealthiest decedents. As seen previously for France in Figure 2, top wealth groups include a significant presence of younger individuals. Since mortality rates are approximately exponential in age, the impact of these younger individuals on the most representative multiplier for decedents is, in fact, substantial.<sup>6</sup>

<sup>6</sup>This is a direct implication of Jensen’s inequality for the exponential function:

$$\text{Exp}[E[a]] < E[\text{Exp}[a]] , \quad (\text{B.1})$$

where  $a$  is decedent age. Because the multipliers depend exponentially on age, the multiplier corresponding to the average age at death is much lower than the average multiplier. Had the dependence of the mortality rate on age been linear, for example, the two quantities would have been equal.

## C The concentration of estates at the top

As shown by [Alvaredo, Atkinson and Morelli \(2018\)](#), relying on unadjusted tax data on estate value can also be informative about the concentration of wealth at the top. To show that, we first need to define the top estate share of quantile  $q$

$$(1 - L_q)^E = \frac{\sum_{i=1}^{qN_E} w_{E,i}}{W_E}. \quad (\text{C.1})$$

This requires summing the estates of the richest  $qN_E$  decedents and estimating the total value of estates of the full decedent population,  $W_E$ . However, the estimation of the latter is not a trivial exercise. It requires the estimation of the total value of unobserved estates of the deceased excluded from the tax records,  $W_E^{exc}$ . This creates uncertainty in the top estate share estimates.

In practice, estimating  $W_E^{exc}$  can be done using the total wealth of the living population not represented by the re-weighted tax records (excluded population),  $N^{exc} = N - \sum_{i=1}^{N_E} m_i$ . The latter can be directly estimated from external sources of data, such as surveys or other administrative records, if the general identity of the excluded population could be inferred.

The total identified wealth is known through the multipliers and observed estate values:

$$W^{iden} = \sum_{i=1}^{N_E^{tax}} m_i w_{E,i}. \quad (\text{C.2})$$

In the absence of heterogeneous multipliers this becomes

$$W^{iden} = \sum_{i=1}^{N_E^{tax}} \bar{m} w_{E,i} = \bar{m} W_E^{iden}. \quad (\text{C.3})$$

The total excluded wealth is then

$$W^{exc} = W - W^{iden}. \quad (\text{C.4})$$

At the same time

$$W^{exc} = \bar{m}^{exc} W_E^{exc}, \quad (\text{C.5})$$

where  $\bar{m}^{exc}$  is the average multiplier of the excluded decedents.  $\bar{m}^{exc}$  can be estimated depending on how refined the demographic data and mortality data available are. If mortality by age and gender is available, it is possible to define a different multiplier for the excluded decedents in each age and gender:

$$m_{a,g}^{exc} = \frac{N_{a,g}^{exc}}{N_{E,a,g}^{exc}}, \quad (\text{C.6})$$

where  $N_{a,g}^{exc}$  is the number of living with age  $a$  and gender  $g$  not observed by the tax records, and  $N_{E,a,g}^{exc}$  is the number of decedents with age  $a$  and gender  $g$  not observed by the tax records. In this

case  $\bar{m}^{exc}$  would be the average of all multipliers  $m_{a,g}^{exc}$ . Alternatively, in the absence of such data,  $\bar{m}^{exc}$  can be defined as the ratio between the excluded living population and the excluded decedent population:

$$\bar{m}^{exc} = \frac{N^{exc}}{N_E - N_E^{tax}}. \quad (\text{C.7})$$

It is clear that different sets of multipliers would lead to different estimates of  $W_E^{exc}$ . This leads to different total values of estates, which, in turn, leads to different top estate share estimates. In Sec. 3 we use this calculation to provide different estimates of top estate shares and compare them to top wealth shares reported in the literature.

## D Mortality rates by age

Age is the most important statistical determinant of mortality. Figure 7 shows the mortality rates in France, Italy, the United Kingdom, and the United States in 1950, 1970, 1990, and 2010, based on [The Human Mortality Database \(2018\)](#) data. It illustrates that mortality rates increase exponentially with age above the age of 40.

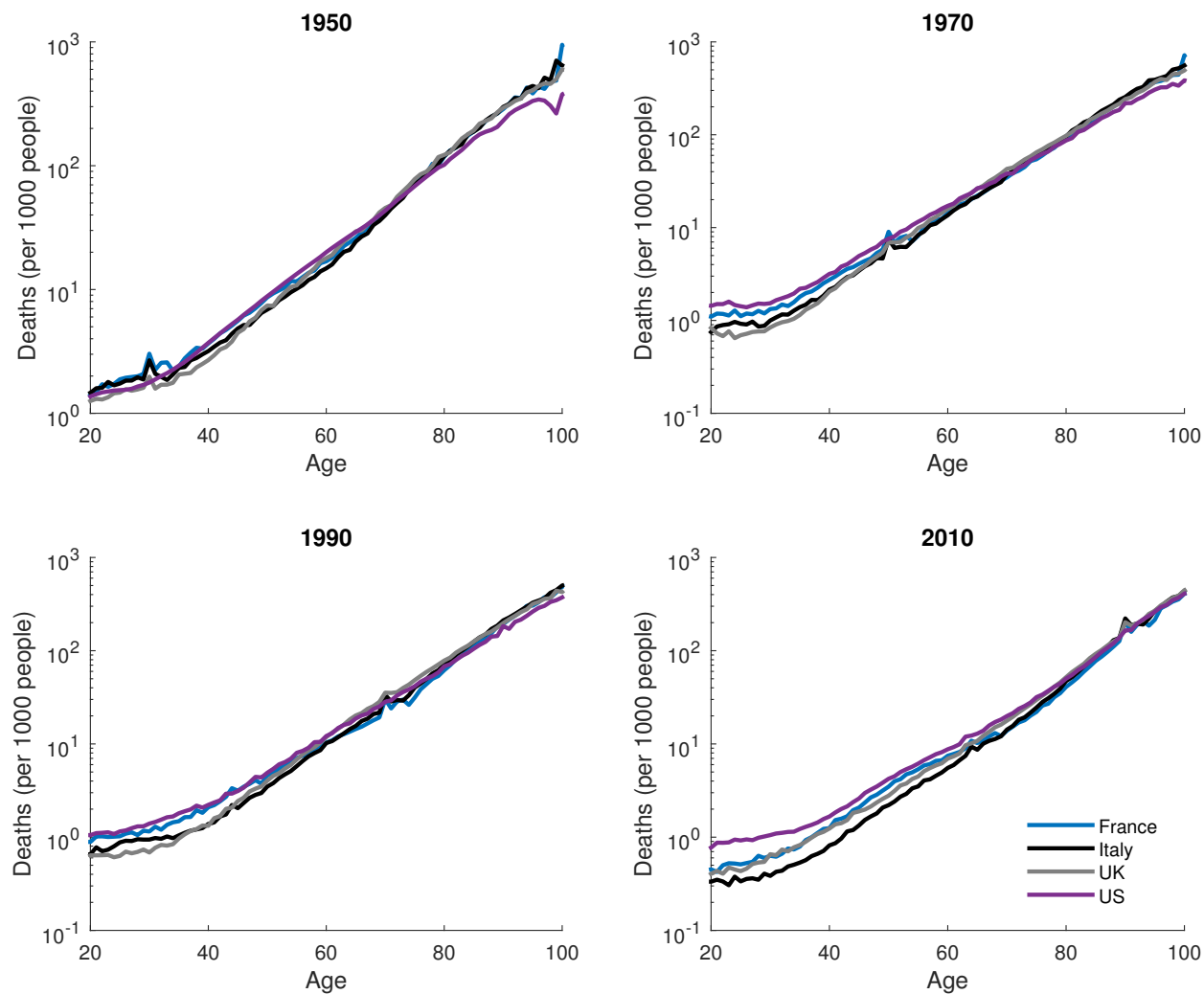


Figure 7: Mortality rates in France, Italy, the United Kingdom, and the United States in 1950, 1970, 1990, and 2010. Source: [The Human Mortality Database \(2018\)](#).

## E The coefficient of variation of estates and of wealths

To illustrate the similarity between the concentration of wealth and of estates it is possible to compare the coefficient of variation (CV) of the wealth distribution with and without multipliers. The derivation is inspired by a derivation presented in [Atkinson and Harrison \(1978\)](#), comparing the CV between capital income and wealth for the capitalization method. It clarifies the intuition for the result obtained for top shares discussed above. Yet it is conceptually simpler, since the index  $I_q$  does not play a role in the CV. It is also not limited to a specific quantile  $q$ , but involves the entire distribution.

The coefficient of variation of estates, denoted  $Y_E$ , follows

$$Y_E^2 = \frac{\sigma_E^2}{\bar{w}_E^2}. \quad (\text{E.1})$$

The coefficient of variation of wealths, denoted  $Y_W$ , follows

$$Y_W^2 = \frac{\sigma_W^2}{\bar{w}_W^2}, \quad (\text{E.2})$$

where  $\sigma_E^2$  is the variance of estates,  $\sigma_W^2$  is the variance of wealths,  $\bar{w}_E$  is the average estate, and  $\bar{w}_W$  is the average wealth.

We begin by writing down expressions for the variance estates and wealths:

$$\sigma_E^2 = \frac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}^2 - \frac{1}{N_E^2} \left( \sum_{i=1}^{N_E} w_{E,i} \right)^2; \quad (\text{E.3})$$

$$\sigma_W^2 = \frac{1}{N} \sum_{i=1}^{N_E} m_i w_{E,i}^2 - \frac{1}{N^2} \left( \sum_{i=1}^{N_E} m_i w_{E,i} \right)^2. \quad (\text{E.4})$$

Therefore we get

$$Y_E^2 = \frac{\frac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}^2 - \frac{1}{N_E^2} \left( \sum_{i=1}^{N_E} w_{E,i} \right)^2}{\frac{1}{N_E^2} \left( \sum_{i=1}^{N_E} w_{E,i} \right)^2}, \quad (\text{E.5})$$

and

$$Y_W^2 = \frac{\frac{1}{N} \sum_{i=1}^{N_E} m_i w_{E,i}^2 - \frac{1}{N^2} \left( \sum_{i=1}^{N_E} m_i w_{E,i} \right)^2}{\frac{1}{N^2} \left( \sum_{i=1}^{N_E} m_i w_{E,i} \right)^2}. \quad (\text{E.6})$$

$\mu$  is the ratio between the average estate and the average wealth

$$\mu = \frac{\frac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}}{\frac{1}{N} \sum_{i=1}^{N_E} m_i w_{E,i}} = \bar{m} \frac{\sum_{i=1}^{N_E} w_{E,i}}{\sum_{i=1}^{N_E} m_i w_{E,i}}, \quad (\text{E.7})$$

so

$$\frac{1}{N^2} \left( \sum_{i=1}^{N_E} m_i w_{E,i} \right)^2 = \frac{1}{\mu^2} \cdot \frac{1}{N_E^2} \left( \sum_{i=1}^{N_E} w_{E,i} \right)^2, \quad (\text{E.8})$$

and therefore

$$Y_W^2 = \frac{\frac{1}{N} \sum_{i=1}^{N_E} \mu^2 m_i w_{E,i}^2 - \frac{1}{N_E^2} \left( \sum_{i=1}^{N_E} w_{E,i} \right)^2}{\frac{1}{N_E^2} \left( \sum_{i=1}^{N_E} w_{E,i} \right)^2}. \quad (\text{E.9})$$

We can then rearrange  $Y_W^2$  and get

$$Y_W^2 = Y_E^2 - \frac{\frac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}^2}{\frac{1}{N_E^2} \left( \sum_{i=1}^{N_E} w_{E,i} \right)^2} + \frac{\frac{1}{N} \sum_{i=1}^{N_E} \mu^2 m_i w_{E,i}^2}{\frac{1}{N_E^2} \left( \sum_{i=1}^{N_E} w_{E,i} \right)^2}. \quad (\text{E.10})$$

Taking  $N = \bar{m} N_E$  we get

$$Y_W^2 = Y_E^2 \left( 1 + \frac{\frac{1}{N_E} \sum_{i=1}^{N_E} \left( \frac{\mu^2 m_i}{\bar{m}} - 1 \right) w_{E,i}^2}{\sigma_E^2} \right). \quad (\text{E.11})$$

This result leads to several important observations that clarify the similarity between inequality of estates and of wealth. First, the difference between the CV of wealth and estates is mainly driven by the multipliers at the top of the distribution. This is because the difference  $\left( \frac{\mu^2 m_i}{\bar{m}} - 1 \right)$  is weighted by the level of estates. Thus, the similarity between  $Y_W$  and  $Y_E$ , like the top shares, mainly depends on the interaction between estates and multipliers among the richest decedents.

Second, there is a dampening effect that limits the extent to which  $Y_W$  and  $Y_E$  are distant from one another. If the multipliers at the top are high in comparison to the average multiplier then  $m_i/\bar{m} > 1$ .  $\mu$  is then likely to be lower than 1. The inverse is true if  $m_i/\bar{m} < 1$ . This creates a dampening effect that makes the expressions  $\left( \frac{\mu^2 m_i}{\bar{m}} - 1 \right)$  in Eq. (E.11) generally close to 0.

Third, comparing the coefficients of variation further demonstrates that the similarity in inequality measures between estates and wealth may not be limited to top shares, but also when full distributions are taken into account.