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ABSTRACT

Our paper explores a transmission mechanism of monetary policy through bond market. Based on the assumption of delayed responses of economic agents to monetary shocks, we derive a system of equations relating the term structure of interest rates with the past history of money growth rates and test the equations with the US data. Our results confirm that the higher ordered moments of money growth rate(converted from the past history of money growth rates) influence the yields of bonds with various maturities in different timing as well as in different magnitudes and monetary policy targeting a certain shape of the term structure of interest rates could be implemented with certain time lags due to path-dependency of interest rates.

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I. Introduction

The primary purpose of our paper is to investigate the roles of monetary policy in shaping the term structure of interest rates. The roles of money are defined in various ways. Among the most critical ones are the roles of money as an accounting unit, a store of value, and a medium of exchange. Due to these particular functions of money, monetary policy governing the stock of money, influences the relative prices of money delivered at different time and different states. In turn, the current relative prices of money to deliver at different points of time in the future, which are, in other words, collectively called the term structure of interest rates, influence economic decisions of private agents. Thus, a thorough exposition of monetary policy would encompass the analysis of a monetary general equilibrium model including all of the above mentioned processes. However, for tractability, we narrow down the scope of this paper to demonstrating how the monetary policy can manipulate diverse interest rates along the passage of time.

Intuitively speaking, the term structure of interest rates is much more informative than any set of economic variables and thus will be useful as a reference for monetary policy. So far there have been continuous debates over what should be optimal targets of monetary

policies. Mostly a combination of inflation and GDP gap is cited as a candidate for the target of monetary policy (Taylor(1993)). Further developed models would allow autoregressive formations in inflation and GDP gap (Clarida, Gali, and Gertler(2000)). Based on such criteria, a certain level of short-term interest rate (e.g. call rate in Korea, federal fund rate in the US) is prescribed that a central bank should maintain. Though such concentration on the determination of the short-term interest rate is relatively easy to implement in practice, it only sequentially cross-checks the level of inflation and GDP gap with the current short-term interest rate. It neglects how the term structure of interest rates as a whole reacts to the adjustment of the short-term interest rates, which might explain why the same level of the short-term interest rate brings about different economic performances at different time and states.

Frequently we read numerous articles about predicting the future path of federal fund rate from newspapers. All of them are written on the implicit belief that monetary policy has influence on major aggregate economic activities, such as consumption, investment, and production, though its influence on these economic activities may differ in terms of directions, magnitudes, and timing. Unfortunately, a true transmission mechanism of monetary policy has not yet been thoroughly explored. A true description for the economy

would be that the transmission mechanism works through multi-channels, only a small number of which so far have been highlighted. In our knowledge, very few economic models have emphasized the differential time effects of monetary policy in the context of analyzing the movements of the whole nominal bond market equilibrium¹.

Apart from the tradition, our paper argues that an effective monetary policy should consider the whole term structure of interest rates rather than a yield rate of a bond with specific maturity. Furthermore, though control over the short-term interest rate has influence on the yields of bonds with longer maturities, it has not yet been clearly verified in which direction a change in the short-term interest rate shifts the whole term structure of the interest rates. Considered that different yield curves lead to different performances of an economy, the monetary authority should perceive at least the impact of its current short-term interest rate policy on the term structure of interest rates. However, an answer to

¹ Most of the literature assumes that the shape of the term structure curve depends on the anticipation for the future, the formation of which is hard to define or requires a somewhat arbitrary mechanism. For example, Ellingsen and Söderström (2004) explain how the yield curve responds to monetary policy. In their work, monetary policy is determined by the central bank's preference parameters over the volatilities of inflation, output, and the short-term interest rate. They claim variations in the preferences result in another yield curve by affecting people's expectation for the future. In contrast, our paper focuses on verifying the relationship between the yield curve and the past money growth rates.

this question would require thorough understanding of the whole economy as well as the bond market itself.

Most of economic activities are determined by the anticipation of the future, which is well embedded in the term structure of interest rates. Furthermore, the shape of the yield curve controlled by the money growth rates or the short-term interest rate does a crucial role in determining the levels of the economic activities. Thus, we are interested in exploring how money growth rate or short-term interest rate policy shifts the term structure of interest rates.

From the literature on durable consumption and investment, we understand that both of them are quite sensitive to economic fluctuations in comparison with consumption on non-durable goods and services. Intuitively speaking, since the flows of benefit from durable goods and capital continue for a certain period of time, durable good consumption and investment entail the feature of irreversibility or indivisibility of purchase, which reduces durable goods consumption and investment decisions to optimal stopping problems. Hence, it is absurd to expect that the monetary authority can raise aggregate demands for durable goods and physical capital by merely changing the short-term interest rate. It is because in reality the falling short-term interest rate is often accompanied by an increase in

the long-term interest rate, which may discourage an agent from purchasing durable goods and physical capital. Thus, the monetary authority should find a certain pattern of a yield curve in order to reset the current yield curve to the pattern, which will boost the aggregate demand in times of depression.

On the other hand, supply side is also dependent on the term structure of interest rates. Production requires a multi-period binding planning horizon in addition to a time-to-build capital driven technology, in which the adjustments of production inputs are not completely flexible across time. Thus, the assignment or the employment of production inputs, not only capital but also labor, is perceived to be a function of the term structure of interest rates.

Our paper proposes to (1) investigate how a monetary policy (not only quantity-easing but also targeted at controlling the short-term interest rate) shifts the whole term structure of interest rates, (2) discuss the implications of the observations that production as well as durable consumption and investment are sensitive to changes in the term structure of interest rates, and (3) arrange monetary policies of maintaining a certain shape of the term structure of interest rates based on empirical results.

The contents of the paper are organized as follows: Section 2 discusses a transmission channel of monetary policy in the economy, which relies on the lagged adjustment

processes of various interest rates in the bond market. The feature of lagged adjustments resulting from delayed responses to monetary shocks is critical in that it relates the dynamics of interest rates to the past history of money growth rates. Section 3 tests all the hypotheses obtained from the models introduced in section 2 using the US data, both monthly and quarterly. The relationship between the term structure of interest rates and the money growth rates are estimated in presence of as well as in absence of endogenous production fluctuations. Section 4 deduces the policy implications by discussing the time lags of monetary policy in implementing a certain yield curve as well as considering the impact of the current short-term interest rate targeting policy on the yield curve. Finally section 5 concludes.

II. Theoretical Framework

From a survey of the current literature on the optimal monetary policy, we identify two common approaches from two distinctive traditions of thoughts-new classical and new Keynesian. New classical approach² admits that market incompleteness, such as market segmentation, may cause the differential effects of monetary policy across time and across agents in the short run whereas new Keynesian approach³ introduces sticky prices and wages to refute the neutrality of money. Regardless of different appearances, these two approaches have common in that they assume private agents respond to shocks in heterogeneous ways.

This section is purposed to provide a logical explanation about the delayed responses of aggregate macro variables to monetary shocks and reveal the consequences of the delayed responses on the dynamics of the term structure of interest rates induced by monetary policy. From the perspective of new classical approach, we build a model, which allows a path dependent dynamics of the interest rates governed by the past money growth rates.

To begin with, we investigate a limited bond market participation model and show that

² Refer to Alvarez, Lucas and Weber (2001) and Monnet and Weber (2001).

³ For more details, refer to Clarida et. al. (1999) and Yun (1996).

the higher order moments of money supply can influence the term structure of interest rates. Extended from a traditional Cash-in-Advance model of Lucas and Stokey(1987), a general m -period-ahead CIA condition is imposed. The adoption of CIA feature is critical because it, combined with the assumption of limited bond market participation, brings about the more persistent redistribution effects of monetary policy on the economy. Based on the assumptions, the term structure of interest rates is approximated by a system of linear equations of the lagged money growth rates. As is generally understood (Clarida, Gali, and Gertler(2000) and Ellingsen and Söderström(2004)), the expectation of the future money growth rates (or the future monetary policy) has effect on the current term structure of interest rates. However, we emphasize the importance of the past path of monetary expansion in a sense that money shock would be realized in differential manners across heterogeneous agents in the economy.

Second, we explore the implications the non-negativity restriction of nominal bond yield rates holds in financial market, while showing that the linear approximation of the term structure of interest rates by the past money growth path does not necessarily satisfy the non-negative condition. The non-negativity restriction of nominal bond rate is a critical

barrier for the central bank to consider when it exercises open market operation policy. Especially, in a very low inflation regime, the possibility of reaching zero short-term interest rate often cast worries because zero rate is regarded as a natural lower boundary of so called liquidity trap. It is commonly believed that the monetary policy without coordination with the expansionary fiscal policy would be ineffective in such a situation. However, the ineffectiveness of monetary expansion in case of falling into the zero nominal interest rate trap may be supported when only one type of bond is available in the financial market other than money. Such extreme absence of variety in bond market is not realistic at all, and the plunge of the whole term structure into zero has not been observed in the history, either. Hence, after complementing our term structure model with non-negativity restrictions, we discuss the effectiveness of monetary policy near zero short-term interest rate and explore a transitional path on which the bond market equilibrium retrieves the positivity of interest rates.

Third, we examine a claim that consumption, investment, and production decisions are significantly affected by the term structure of interest rates while the demands for durable goods and production factors are more sensitive to a change in the term structure of interest rates than consumption of non-durable goods and services due to their (longer) duration of

usage. All other things equal, a lower short-term interest rate is likely to induce more current consumption. However, what if the lowered short-term interest rate is matched by higher long-term interest rate? An answer without considering the dynamics of the term structure would lead to the imprecise reasoning that lowering the short-term interest rate encourages the consumption. Thus, we also aim to answer for a question why a monetary policy targeting a certain level of the short-term interest rate leads to different economic performances at different times and states.

Nominal bonds, which guarantee the delivery of pre-defined amount of money at maturities, are (gross) substitutes for money⁴. Private agents allocate their resources between money and nominal bonds⁵. Hence, a change in money stock indicates that the economy should move to another equilibrium sustaining different relative prices of bonds with respect to money. This section focuses on analyzing a mechanism, through which variations in monetary policy lead to different term structure of interest rates. A basic idea that the past money growth path determine the current term structure of interest rates, would explain why it leads to different outcomes to maintain the same level of the short

⁴ In other words, money is a kind of nominal bond, which expires and is renewed instantly.

⁵ In fact, nominal bonds vary not only by the length of maturities but also by the magnitude of default risk. However, for simplicity our paper deals with government issued bonds only. The status of the government as a sole provider of currency in the economy eliminates default risk premium on the government bonds.

term interest rate at different periods⁶

Needless to say it would be another paper topic to verify whether and how the term structure of interest rate can have real effects on the economy. The emphasis on the relationship of the term structure of interest rates and real macro variables is originated from our original intention to transform the issue of finding optimal monetary policy to that of finding an appropriate term structure, which induces more consumption, investment and production. However, in this paper, we do not delve into this issue further. Instead we concentrate on revealing the relationship between money growth rates and the term structure of interest rates.

II-1. Lagged transmission channel of monetary shocks

In this section we derive an equation linking the term structure of interest rates with the past history of money growth rates. We introduce an economy with limited bond market participation in order to induce a situation in which a monetary shock has differential

⁶ Of course, it is reasonable that the yield curve is also influenced by the expectation of the future money growth rates and we need a model where the term structure of interest rates depends on the future monetary policy as well as the past history of monetary policies. However, we have no clear clue as to how the accumulation of the information on the past history is reflected on the formation of the expectation for the future. Thus, instead of the future variables being separately included, the expectation for the future can be understood as reflection of the past history. In this sense the persistent

impacts on heterogeneous agents across time (mainly redistribution effects). The impact differentials are caused by the unsynchronous timing of money shock transmitted to or perceived by the agents or by their different speed of reactions to the shock, and they lead to a non-trivial change in the term structure of interest rates. On the other hand, in absence of such impact differentials, the yield curve would shift up or down in parallel according to the change of the present and the past money growth rates. A swing of the yield curve would be possible only by the coordinated variations of the expectation for the future monetary growth path and other real macro variables.

Our model is an adapted version of Alvarez et al.(2001). Our model assumes the following. First, there are two types of assets in the market-money and bond. Considered that the assets are means of storing or growing values along the passage of time, the nominal return on money is always zero by construction whereas the nominal return on bond is positive nominal interest rate. Due to the yield difference in these two types of assets, we need a mechanism guaranteeing the positive holding of money. Thus, we assign a CIA restriction, which is modified from the original one in Lucas and Stokey (1987).

Second, we assume limited bond market participation, under which not every consumer

effect of the past policy can be more substantial than we guess

can purchase bonds in the financial market due to transaction costs or information costs or regulation. There are two groups of consumers in the market-bond market participants and non-participants, whose shares in the total population are \square and $1-\square$ respectively⁷. These two groups are homogeneous in all the other aspects than the bond market participation.

Third, the CIA condition to be introduced is defined on a multi-period time horizon as follows. At the current period, nominal consumption is afforded by a certain portion from the current nominal income, another certain portion from nominal income of the previous period, another certain portion from income earned two period ago, and so on. A more intuitive interpretation of the multi-period ahead CIA condition is that at the beginning of period t the current income (y_t) would be cashed instantly ($p_t y_t$) and it would be spent for the next m periods by certain fractions of $v_{t,t+j}$, $j = 0, 1, 2, \dots, m-1$, ($\sum_{j=0}^{m-1} v_{t,t+j} = 1$).

Based on the above model, we derive a system of equations of our concern as below⁸.

$$\Gamma_t = \Phi \Delta_t + R(v^t, g^t) + \varepsilon_t, \quad (1)$$

⁷ It is assumed that all the bond market participants hold all kinds of bonds with various maturities. A more realistic setup would allow that the bonds market participants should be classified into several groups by the maturities of bonds they hold (for example, short-term, medium-term, and long-term investors). Then, then the equilibrium yield rate would display more dynamism.

⁸ For more details on the derivation of the equations, see Appendix A. In Appendix A, we derive the system of equations with additional simplifying assumptions, such as zero GDP growth rate ($g_t = 0$ for all t) and the absence of taxation ($\tau_t = 0$ for all t). In contrast Equation (1) covers more general cases.

$$\Gamma_t \equiv \begin{bmatrix} r_{t,t+1} \\ r_{t,t+2} \\ \dots \\ r_{t,t+n-1} \\ r_{t,t+n} \end{bmatrix}, \quad \Delta_t \equiv \begin{bmatrix} \mu_t \\ \mu_{t-1} \\ \dots \\ \mu_{t-m+2} \\ \mu_{t-m+1} \end{bmatrix}, \quad \Phi \equiv \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} & \dots & \phi_{1,m-1} & \phi_{1,m} \\ \phi_{2,1} & \phi_{2,2} & \dots & \dots & \dots & \phi_{2,m} \\ \phi_{3,1} & \dots & \dots & \phi_{i,j} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \phi_{n-1,m} \\ \phi_{n,1} & \phi_{n,2} & \dots & \dots & \phi_{n,m-1} & \phi_{n,m} \end{bmatrix},$$

where Γ_t is a $n \times 1$ vector of yield rates with different maturities, Δ_t a $m \times 1$ vector of money growth rates up to date for the last $m-1$ periods, R a $n \times 1$ vector, and Φ a $n \times m$ matrix. $R(v^t, g^t)$ is the term evaluating the effects of other variables on the term structure of interest rates, such as a vector of the current and the past GDP growth rates (g^t) and is closely related to the current and the past velocities of money circulation (v^t)⁹. The importance of $R(v^t, g^t)$ is highlighted later in empirical analysis.

The above equations show path dependency in that the present term structure of interest rates is affected not only by the money growth rate of the current period but also by those of the past ($m-1$) periods¹⁰. Theoretically, path dependency is a common phenomenon and may arise from various sources. First, it can come from the learning process. All the economic decisions in a dynamic context should involve the formation of expectation for the future, which is in turn based on the learning process from the past experience. This is also an excuse for not including the expectation for the future in the model. Second, path dependency can arise from some sort of market frictions, which prevent economic agents from responding to shocks in a uniform manner and with simultaneous timing. Such inevitably heterogeneous responses of the agents may lead to persistent and lagging effects of monetary policy. There are many other sources of path dependency, but here we are particularly interested in these two sources. Our paper introduces frictions in

⁹ For formal definitions of g^t and v^t , see Appendix A.

¹⁰ Money growth rates for the past $m-1$ periods can be replaced by the higher order moments of the money growth rate (μ_t) up to $m-1$ th order.

consumer/investor side in order to derive a path dependent relation of interest rates and money growth.

Another notable point from Equation (1) is that the lagged adjustments of interest rates in response to monetary policy vary across different types of bonds in terms of directions as well as magnitudes of changes. This implies that the monetary authority can adjust the shape of the term structure by using the dynamic or path dependent relation of the term structure with monetary policy. As earlier mentioned, understanding the dynamics of the term structure is very important because most major economic activities, such as durable consumption and investment, are significantly influenced by the shape of the term structure. However, to find an optimal term structure is beyond the scope of this paper. Instead we focus on how a certain term structure of interest rates could be implemented with the accommodation of monetary policy.

II-2. Zero lower boundary and liquidity trap

The term structure of interest rates described in Equation (1) provides static information evaluated at a point of time on the dynamics of various interest rates. Considered that Equation (1) is obtained from the first order log-linear approximation of Equation (A-2), the interest rate dynamics may violate the non-negativity of nominal interest rates and the

non-negativity restrictions should be additionally levied on the yields of all maturities.

A nominal interest rate is the rate of return on holding nominal bonds. Due to the definition and the existence of money, zero is a natural lower boundary for the nominal interest. So far the probability of hitting zero interest rate has been evaluated extremely low and the consideration of non-negativity yields has not been strongly enforced. However, the recent low interest rate regime in a few economies including US and Japan has caused worries that the nominal interest rate might hit zero and the economy might fall into the natural lower bound of the liquidity trap.

In this section, we analyze the propagation mechanism of the monetary policy in case of hitting the zero short-term interest rate by levying the non-negativity restriction on Equation (1). In addition, we distinguish the liquidity trap from the state of zero nominal interest rate and discuss an escape strategy from each of them using monetary policy.

There may be various ways of assigning the non-negative condition to Equation (1). Among them, the most intuitive one is to introduce shadow processes, which are equivalent with the yield rates when they are positive and diverges (become negative) when the yield rates are zero. In consideration of the non-negativity condition as above, Equation (1) should be modified to

$$\begin{aligned}
\begin{bmatrix} r_{t,t+1} \\ r_{t,t+2} \\ \vdots \\ r_{t,t+n-1} \\ r_{t,t+n} \end{bmatrix} &\equiv \begin{bmatrix} \max \left[\sum_{j=1}^m \phi_{1,j} \mu_{t-j}^E + R_1(v^t, g^t) + \varepsilon_{1t}, 0 \right] \\ \max \left[\sum_{j=1}^m \phi_{2,j} \mu_{t-j}^E + R_2(v^t, g^t) + \varepsilon_{2t}, 0 \right] \\ \vdots \\ \max \left[\sum_{j=1}^m \phi_{n-1,j} \mu_{t-j}^E + R_{n-1}(v^t, g^t) + \varepsilon_{n-1t}, 0 \right] \\ \max \left[\sum_{j=1}^m \phi_{n,j} \mu_{t-j}^E + R_n(v^t, g^t) + \varepsilon_{nt}, 0 \right] \end{bmatrix} \quad (2) \\
R(v^t, g^t) &\equiv \begin{bmatrix} R_1(v^t, g^t) \\ R_2(v^t, g^t) \\ \dots \\ R_{n-1}(v^t, g^t) \\ R_n(v^t, g^t) \end{bmatrix} \quad \text{and} \quad \varepsilon_t \equiv \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \dots \\ \varepsilon_{n-1t} \\ \varepsilon_{nt} \end{bmatrix}
\end{aligned}$$

Looking at Equation (2), we may wonder what difference it makes from Equation (1) except the additions of an operator $\max [x,0]$ to each row. A more critical difference could be found in the movement of a newly defined money growth rate μ_t^E . μ_t^E is defined to be the effective money growth rate and is equal to the pre-defined money growth rate μ_t in absence of a zero rate bond. The divergence of μ_t^E from μ_t arises when the yield rate of a bond hits, or stays at, or escapes from the zero boundary. It is because a bond, once its yield rate hits zero, would be treated as an equal for money. Accordingly, the money growth rate should be modified to account for a sudden change in the categories of

money stock. Likewise, when the bond yield escapes from the zero rate, the exactly opposite movement in the money growth rate as well as in the money stock would be observed.

So far we haven't clarified how the zero short-term interest rate is different from the liquidity trap. The liquidity trap is a state in which monetary expansion through open market operations or helicopter money drops cannot encourage economic agents to increase bond holdings and lower the interest rate further. In other words, the liquidity trap is a mental phenomenon, in which the substitution between money and bonds is extremely sensitive to the interest rate change. Accordingly, the level of the short-term interest rate, at which the liquidity trap arises, doesn't have to be zero.

On the other hand, the zero short-term interest rate does not necessarily imply the advent of the liquidity trap. There has never been a period in which the whole term structure collapsed into the zero line, though there were some cases in which a point on the term structure curve hit zero. Hence, even in the (near) zero short-term interest rate environment, the monetary authority can carry out expansionary monetary policy through open market operation by using other bonds with positive yield¹¹.

¹¹ Orphanides(2003) appreciates the usefulness of the open market operation policy, which is to "implement additional

Comprehension of the differences between the liquidity trap and the zero interest rate gives a clue to finding escape strategies from the liquidity trap. One of them is to use the increment of money stock neither for tax reduction, nor for the purchase of bonds, but for the purchase of goods. This can be regarded as a fiscal policy in that it increases the government expenditure. On the other hand, it still holds a feature of a monetary policy in that there is no additional fiscal burden in the government account. The inflationary effect of the government expenditure expansion funded by printing money would induce private agents to consume more and faster. In other words, the inflationary policy raises the velocity of money $(\frac{1}{1 - v_{t,t}})$. The faster velocity is exactly opposite to the common belief that monetary expansion through the open market operation reduces the velocity of money in a liquidity trap. A more detailed description of the escape strategy is available in Appendix A.

II-3. Consumption, investment, production and the term structure of interest rates

In this section we discuss the relationships of the term structure of interest rates with consumption, investment and production. The term structure of interest rates matters

monetary expansion by shifting the targeted interest rates to that on successively longer-term instruments, when additional monetary policy easing is warranted at near zero interest rates".

because most intertemporal decisions are the functions of the term structure of interest rates. Among the various intertemporal decisions made by economic agents, we are particularly interested in consumption on durable goods and capital investment as well as production because all of them take substantial portion in the economy and they are more volatile than other economic decisions¹²

Unlike consumption on non-durable goods and services, capital investment and durable good consumption show more fluctuations in response to economic shocks including interest rate changes. By nature, decisions on durable good consumption and physical capital investment are very close to discrete choice or optimal exercises of real options in presence of indivisibility and irreversibility¹³. To rephrase, durable good consumption and capital investment are simply reduced to optimal stopping models, in which the term structure of interest rates is a critical determinant.

Hong(1996) and Hong(1997) compare the sensitivities of durable good consumption and fixed capital investment to price and interest rate changes using the US data and shows that

¹² Consumption on durable goods and capital investment constitute aggregate demand whereas production determines the aggregate supply of an economy.

¹³ Due to concavity of instantaneous utility functions, an agent prefers to smooth cross-time allocation of consumption. Thus, he prefers to schedule consumption on both durable and non-durable goods evenly across time. On the other hand, consumption of durable goods is measured by the stock of the durable goods accumulated up to date and the change in the consumption on durable goods is net purchase of durable goods at the current period. Accordingly, the net purchase of durable goods is more volatile than consumption of durable goods in order to guarantee the smoothing of durable good consumption. This is another reason that the purchase of durable goods draws more attention in diagnosing a business

durable good consumption and investment react sensitively to the price change but not so sensitively to the variations in the interest rate. He interprets that the price, which reflects the longer horizon forecast of an economy, is more influential in determining durable goods consumption than the short-term interest rate. The linkage of his idea to this paper is in that the price of durable good is the discounted sum of the future benefit flows by the term structure of interest rates. In addition, Breitung, Chrinko, and Kalckreuth (2003), using German firm data, reach a similar conclusion that business investment is responsive to the user cost of capital.

Summing up, the private agents make decision based on both the future cash flows and the interest rate movement. The future path of interest rates, anticipated from the yield rates of bonds with different maturities, is linked with consumption and investment decisions. Channels, through which monetary policy affects the economy, may be numerous. However, the channel through the bond market is the most direct but the least mentioned one.

Weakness of our model is that it doesn't consider the effect of monetary policy on production. Description of the production sector and its interactions with monetary policy

cycle.

are omitted because the introduction of a production function in the economy would require the calculation of a steady state and discussions on transitional paths. True that the interactions of monetary policy with production is crucial, we do not pursue in the direction further¹⁴. Instead we represent a supply condition by linking the real sector production growth with the term structure of interest rates. In reality, most production inputs, not only physical capital but also labor employment, are more or less irreversible in a sense that commonly the contracts for hiring these production factors are made for a few years in advance. Thus, the current production growth should reflect the past anticipation for the long-run economic forecasts, which is recorded in the past term structure of interest rates. Hence, we accept the supply condition as below:

$$g_t \equiv \frac{y_t - y_{t-1}}{y_{t-1}} = f(r_{t-q,t-q+1}, r_{t-q,t-q+2}, \dots, r_{t-q,t-q+n}) + \eta_t \quad (3)$$

$$g_t = \sum_{i=1}^n s_i r_{t-q,t-q+i} + s_0 + \eta_t$$

¹⁴ As a further extension or a generalization of our model, we may consider the supply side restriction jointly with Equation (1). For a more general setup allowing for delayed responses of producers, an aggregate supply function could be represented as follows: $\pi_t = \sum_{i=0}^u k_i E_{t-i} x_t + \sum_{i=1}^z \varsigma_i \pi_{t-i} + \sum_{i=1}^k \gamma_i E_{t-d} \pi_{t+i}$

where x_t is a real GDP gap at time t (refer to Woodford (2003)).

Equation (3) is different from a usual Phillips curve type supply condition, which describes the relationship between the inflation rate and the real GDP gap. However, the differences are acceptable on the following grounds. First, the concept of potential GDP used in the Phillips curve is ambiguous and it is estimated merely by filtering the real GDP data. Second, the information on the future inflation rates is already embedded in the term structure of (nominal) interest rates. Third, labor supply, a determinant of real GDP gap, is chosen simultaneously with household consumption and it is also influenced by the term structure of interest rates. In the following section, Equation (3) is jointly estimated with Equation (1) in order to eliminate possible endogeneity of interest rate determination arising from running Equation (1) only¹⁵.

III. Empirical Analysis

This section verifies the validity of the claims deduced in the previous section. Equation (1) implies that the term structure of interest rates is governed by the past money growth rates. In this section, mainly we use several modifications of (1) and (3) for empirical analysis.

¹⁵ Intuitively, Equation (1) is a demand condition and Equation (3) is a supply condition for bond market.

III-1. Data

Our analysis is based on the US data from July 1959 to February 2000. We use the US data because the US government bond market is the most developed one and the maturities as well as the volume of the bonds traded in the market are diverse and huge enough to plot a reliable yield curve.

The variables of our concern are money stock, price and income variables in addition to five key interest rates¹⁶. For the key interest rates, we select federal fund rate, 3-month treasury bill, 6-month treasury bill, 1-year treasury bill, and a composite of long-term U.S. government securities¹⁷. For the macro variables, we use M1 for an index of money stock, GDP deflator for price index, and real and potential GDP¹⁸ for income measures.

The data frequencies differ from a category to another. For example, all the interest rates and M1 are recorded monthly whereas GDP deflator and GDP¹⁹ are recorded quarterly. To reconcile the conflicts of the data frequencies at the same time exploiting the benefit of using monthly data, we run models separately with monthly and quarterly data.

¹⁶ Interest rates are measured in annum whereas M1, GDP deflator, and GDP measures are on a quarterly basis.

¹⁷ The composite of the long-term treasury bonds is specifically defined to be an unweighted average on all outstanding bonds neither due nor callable in less than 10 years.

¹⁸ H-P filtered real GDP is used for potential real GDP.

¹⁹ As for the monthly data, an index of industrial production may be used as a proxy for nominal GDP. In that case, since the monthly GDP deflator is unavailable, CPI or PPI index can be substituted for the GDP deflator.

As a variable for money stock, we use seasonally adjusted M1 for a couple of reasons. First, we choose M1 because it is a money stock indicator closest to high powered money. Other money stock indicators, such as M2 and M3, are under the less direct control of the monetary authority and are more likely affected by money demand fluctuations. M1, like other money stock variables, are still susceptible to money demand fluctuations. Admitted that it is hard to distinguish money demand shock from supply shocks, we still maintain the use of M1 because M1 fits much better than the high powered money with the real data.

Second, the data for M1 are seasonally adjusted, considering that the asset prices tend to have no seasonality due to the prevalence of no-arbitrage condition. Accordingly, in order to couple the interest rates with the money growth rates, it is recommendable to use the seasonally detrended M1.

III-2. Test strategies and stationarity of variables

Before running regressions on Equation (1) with or without Equation (3), we test the stationarity of each variable included in the equations by DF-GLS method. The result shows that real GDP growth rate, potential GDP growth rate, and M1 growth rate are stationary with the significance of 1%-10% for the varying lags from 1 to 10. On the other

hand, the velocity of money circulation(v_t), the inflation rate (π_t , measured by GDP deflator) and the yield rates (Γ_t) turn out to be non-stationary.

The stationarity test results indicate that Equation (1) is not testable with the yield rates and the money growth rate only. The remainder $R(v^t, g^t)$ should be a non-stationary process by construction. Hence a test strategy for Equation (1) is either to take the difference for the elimination of non-stationarity or to use $R(v^t, g^t)$ in the estimation procedure by representing it in a linear function of (v^t, g^t) .

Given that the GDP data is not available monthly, only the first strategy is applicable to the monthly data whereas the quarterly data can implement even the second one. Thus, depending on the frequency of the data, we adopt different testable equations. For the monthly data, we use the difference method as below

$$\begin{aligned}
\Gamma_t - \Gamma_{t-1} &= \Phi \Delta_t - \Phi \Delta_{t-1} + R(v^t, g^t) - R(v^{t-1}, g^{t-1}) + \varepsilon_t - \varepsilon_{t-1} \\
&= \Phi(\Delta_t - \Delta_{t-1}) + R(v^t, g^t) - R(v^{t-1}, g^{t-1}) + \varepsilon_t - \varepsilon_{t-1} \\
&= \Phi^* \Delta_t^* + R(v^t, g^t) - R(v^{t-1}, g^{t-1}) + \varepsilon_t - \varepsilon_{t-1} \\
&= \Phi^* \Delta_t^* + \eta_t,
\end{aligned} \tag{4}$$

where

$$\Phi^* \equiv \begin{bmatrix} \phi_{1,1} & \phi_{1,2} - \phi_{1,1} & \phi_{1,3} - \phi_{1,2} & \cdots & \phi_{1,m} - \phi_{1,m-1} & -\phi_{1,m} \\ \phi_{2,1} & \phi_{2,2} - \phi_{2,1} & \cdots & \cdots & \cdots & -\phi_{2,m} \\ \phi_{3,1} & \cdots & \cdots & \phi_{i,j} - \phi_{i,j-1} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & -\phi_{n-1,m} \\ \phi_{n,1} & \phi_{n,2} - \phi_{n,1} & \cdots & \cdots & \phi_{n,m} - \phi_{n,m-1} & -\phi_{n,m} \end{bmatrix}, \Delta_t^* \equiv \begin{bmatrix} \mu_t \\ \mu_{t-1} \\ \cdots \\ \mu_{t-m+1} \\ \mu_{t-m} \end{bmatrix}$$

$$\eta_t \equiv R(v^t, g^t) - R(v^{t-1}, g^{t-1}) + \varepsilon_t - \varepsilon_{t-1}$$

On the other hand, for the quarterly data, we use a fully linearized version of Equation (1) as below:

$$\Gamma_t = \Phi \Delta_t + \Psi_v v^t + \Psi_g g^t + \varepsilon_t, \quad (5)$$

where Ψ_v and Ψ_g are vectors of the same dimension with v^t and g^t respectively.

III-3. Results

Equation (4) and (5) consist of several equations and they are to be estimated by seemingly unrelated regression (SUR) in principle. However, in practice SUR usually underestimates the standard errors of estimates. Hence, we run regressions equation by equation with Newey-West estimates of standard deviations instead of SUR.

Equations (4) and (5) are tested with the monthly and the quarterly US data respectively.

Especially, with the quarterly data, we include GDP deflator, real GDP growth, real GDP

and money stock(M1) for the estimation of Equation (5). In addition, a short-term interest rate policy function of a monetary authority as well as a supply condition (Equation (3)) are jointly estimated with Equation (5).

[Figure 1] displays the historical patterns of the yield rates of our concern. Overall the five key interest rates commove but with apparent idiosyncratic fluctuations. Our paper distinguishes itself from other literature in that it represents such term structure dynamics by a common factor of the current and the past money growth rates.

III-3-1. Tests with Monthly Data

We test Equation (4) with a little modification of $\Phi * \Delta_t^*$. Since the lagged money growth rates in Δ_t^* are hard to interpret intuitively, they are replaced by a vector θ_t , which contains the information on the current money growth rate and its higher order moments^{20 21}.

²⁰ The first order moment of the money growth rate is to be called "slope" and the second one is "curvature". Higher order moments than the second one are to be denoted as their matching ordinal numbers.

²¹ The contents of information in θ_t is equalized to those of Δ_t^* by including higher order moments of money growth up to m.

$$\theta_t \equiv \begin{bmatrix} \mu_t \\ \mu_t - \mu_{t-1} \\ \mu_t - 2\mu_{t-1} + \mu_{t-2} \\ \mu_t - 3\mu_{t-1} + 3\mu_{t-2} - \mu_{t-3} \\ \mu_t - 4\mu_{t-1} + 6\mu_{t-2} - 4\mu_{t-3} + \mu_{t-4} \\ \dots \end{bmatrix}$$

The adoption of θ_t ²² changes Equation (4) to

$$\Gamma_t - \Gamma_{t-1} = \Phi^{**} \theta_t + \eta_t, \quad (6)$$

where Φ^{**} is modified from Φ^* so that it can match with θ_t . We estimate Equation (6) by running regressions equation by equation. The variances of the coefficient estimates are estimated by the Newey-West method.

Results from Equation (6) are displayed in [Table 1]. Money growth rate (μ_t) is excluded from the list of explanatory variables due to very low significance. Instead the next three higher order moments, slope, curvature, and the third order moment of money growth rate, are used in the estimation of Equation (6). Our findings include a couple of notable patterns. First, the signs of coefficients change alternatively from negative to positive and positive to negative. Second, the longer the maturity is, the less likely it is to be influenced by the changes in the higher order moments of money growth.

²² On a quarterly basis, [Figure 2] shows how different order moments of money growth rate move in a heterogeneous way, which is also observable on a monthly basis. Another notable point is that the volatilities of the n-th order moments tend to increase with n as is shown in [Table 6].

Reminded that [Table 1] summarizes the linear relation between the first order differences of yield rates and the higher order moments of money growth rate, we need to convert the results of Equation (6) and evaluate directly the impact of money growth rate on the yield rates. [Table 2] shows the liquidity effect is prevalent in the beginning and the Fisher effect shows up at later periods for all of the five key interest rates. Additionally, [Figure 3], a graphical exposition of [Table 2], discovers a couple of interesting points. First, the longer the maturity is, the less responsive the yield rate is to the changes in money growth rate. Second, the longer the maturity is, the shorter time it would take to get out of the liquidity effect. Third, the bonds with different maturities move in the different directions (at period 1) as well as with different magnitudes.

III-3-2. Test with Quarterly Data

As in the case of the monthly data, we modify Equation (5) to

$$\Gamma_t = \bar{\Phi} \theta_t + \psi_v v_{t,t} + \psi_g g_t + \varepsilon_t, \quad (7)$$

where $\bar{\Phi}$ is modified from Φ so that it can match with θ_t . All the components except the current velocity of money ($v_{t,t}$) are omitted due to unobservability. In addition, for simplicity, only the current growth rate (g_t) is used from the vector (g^t). Furthermore,

$v_{t,t}$ is not directly observable. However, from the money equation of $m_t \frac{1}{1 - v_{t,t}} = P_t y_t$, we see that $v_{t,t}$ is a function of money stock, price level and real GDP. Accordingly, a linear combination of money stock, GDP deflator and the real GDP is substituted for $v_{t,t}$.

Results from running Equation (7) are displayed in [Table 3]²³. As in the case of the monthly data, we run regressions equation by equation with Newey-West estimates of standard errors. However, Equation (7) differs from Equation (6) in that money stock, as a determinant of money velocity, is included and the yield rates, not their first order differences, are used as dependent variables. Compared with Equation (6), Equation (7) has greater explanatory power.

In [Table 3], most higher order moments of money growth rate as well as all of the macro variables are significant at a 5% significance level. The negative signs of money stock and money growth rate reveal the presence of the short-term liquidity effect at least in the short-run. Especially, the negative sign of money stock implies that there even exists a scale of economy in monetary policy and a certain money growth rate may lead to a greater change of the interest rates depending on the size of money stock.

Converting the high order moments of money growth into the lagged money growth rates

²³ The results are mostly the same even when a Taylor type short-term rate policy function and /or a supply side

as in [Table 4], we find that in the short run the signs of the estimated coefficients mostly coincide with our theoretical predictions and they supports the short-term liquidity effect at 95% confidence intervals. In contrast, the long-term Fisher effect is not conspicuous. Since the effect from M1 is not included, the negative sign of its coefficient implies that the cross-time and the cross-sectional persistence of the liquidity effect should be longer than it is shown in [Table 4]

[Figure 4] graphically exposes the cross-sectional variations in the term structure of interest rates along the passage of time in response to 1% increase in money growth rates. [Figure 4] shows that the bonds with different maturities move in the same direction but with varying magnitudes. As seen in the monthly data, the longer the maturity is, the less responsive the yield change is.

[Table 5] is a result from running Equation (3) with $q=1^{24}$. Five different yield rates are used in the estimation procedure-federal fund rate, three and six month treasury bills, 1 year treasury bonds and the unweighted average of yields rates of the long-term government bonds (over 10 years). Alternatively changing signs of the coefficients imply that the rising(slope) and concave(curvature) yield curve accompanied by the low short-term

condition like Equation (3) are/is included. Thus, the results from running only Equation (7) are reported.

interest rate(location) spurs production growth.

IV. Policy Implications

From the previous sections, it is demonstrated theoretically and empirically that the impulse response functions of the yield rates with respect to money shocks determine the shape of the term structure of interest rates. Using this property, the monetary authority can implement a certain shape of the term structure of interest rates when there is no exogenous shocks other than changes in money growth rate. Then, what the monetary authority has to concern about are the representability of a certain term structure of interest rates as well as the time lags to take for the implementation.

IV-1. Implementability and time lags

In a type of Equation (5), the dimension of the $n \times m$ matrix Φ determines the representability of the term structure²⁵. If $\dim \Phi$ is no less than the number of bond types available in the market (n), then a certain money growth rate path can lead to an arbitrary term structure of interest rates within m periods. Otherwise, complete

²⁴ Equation (3) fits best at $q=1$.

²⁵ Representing a certain term structure of interest rates doesn't necessarily guarantee the system would stay at the level

representability is not achievable²⁶

An easier criterion for the representability and the time lags of the implementation process is to check an impulse response matrix, which is defined to be a stack of impulse response function values with respect to maturities and time horizon. Define the impulse response matrix Ξ to be a $n \times T$ matrix, where T is an arbitrarily set time horizon (before all the impulse responses completely phase out) and n is the types of bond maturities available in the market. If $n > T$, then the representability of the system is limited to $\dim(\Xi) < n$. If $n \leq T$ and $\dim(\Xi) > n$, then the composite effect of the money growth rates during the last n quarters can represent any arbitrary term structure of interest rates. Thus, we see that at least the horizons of impulse response functions should be longer than the kinds of assets available in the market in order to guarantee the representability. The time lags of implementation, is not easy to answer due to the presence of multiple solutions. However, the higher dimension of Ξ is more likely to raise the likelihood of attaining at a certain term structure of interest rates within a shorter time horizon.

continuously. Stability is another issue to tackle but will not be dealt with further in the paper.

²⁶ In that case, Gaussian least square method would provide a minimum Δ_t^* from solving

$$\min_{\Delta_t} \varepsilon_t' \varepsilon_t = \left[\bar{\Gamma}_t - \Phi \Delta_t - \Psi_v v^t - \Psi_g g^t \right]' \left[\bar{\Gamma}_t - \Phi \Delta_t - \Psi_v v^t - \Psi_g g^t \right], \text{ where } \bar{\Gamma}_t \text{ is a targeted level of the yield curve.}$$

IV-2. Determination of the short-term interest rate

In reality, it is more often the case that monetary authorities use the short-term interest rate rather than the money stock M1²⁷ for a control variable of monetary policy. Especially in the US, it seems that the federal reserve sets the short-term interest rate based on the deviations of inflation and GDP from certain levels²⁸.

$$r_{t,t+1} = \bar{r} + \phi_{\pi}(\pi_t - \bar{\pi}) + \phi_x(y_t - y_t^p)$$

The effect of such monetary policy of the short-term interest rate determination on the yield curve can be analyzed as a brief extension of our model.

Suppose that the short-term interest rate is prescribed by the federal reserve at period t as in the above Taylor type rule. Then, by combining it with Equation (1) and (7), we obtain an autoregressive equation of money growth rate μ_t as follows.

$$\mu_t = \frac{1}{\phi_{1,1}} \left[\sum_{i=1}^m \phi_{i+1} \mu_{t-i} + \psi_v v_{t,t} + \psi_g g_t - \phi_{\pi}(\pi_t - \bar{\pi}) - \phi_x(y_t - y_t^p) - \bar{r} + \varepsilon_t \right] \quad (8)$$

The impulse response functions of the yield rates in regard to such federal fund rate policy can be obtained by representing the series of $\{\mu_{t-j}, j = 0, 1, \dots, m\}$ by Equation (8) and

²⁷ Throughout the whole paper, we implicitly assume that M1 is under the control of monetary authority. However, in reality, M1 is not directly controlled by the monetary authority because variations in the demand side are hardly predictable and the magnitude of the demand side effect is greater than our anticipation. Despite such problems, we do not use monetary base instead of M1 because the money equation does not hold for the monetary base.

²⁸ Taylor (1993) estimates $r_{t,t+1} = 0.04 + 1.5(\pi_t - 0.02) + 0.5(y_t - y_t^p)$ using the US data of 1980s.

plugging them to Equation (7).

[Table 7] provides results from running simultaneously all the equations of Equation (7) with the Taylor rule. Compared with [Table 3], there are some changes in the size of estimates but the qualitative results remain mostly the same. On the other hand, [Table 8], which provides the results from running a simplified version of Equation (8), shows that the Taylor type short-term interest rate rule causes μ_t to move in an autoregressive way. The first and the second lags of μ_t are positive at 1% significance while M1 holds negative sign at the same significance level. The size of lags is chosen from Bayesian Information Criteria(*BIC*).

IV-3. Escape from zero short-term interest rate

Suppose that the yield rate of *n-period* bond, $r_{t,t+n}$, hits(or escapes from) zero at period t . Then the effective money growth rate and money stock would be

$$\mu_t^E \equiv \mu_t + \frac{B_{t,t+n}}{M_t} \quad \text{and} \quad M_t^E \equiv M_t + B_{t,t+n}, \quad (\text{or} \quad \mu_t^E \equiv \mu_t - \frac{B_{t,t+n}}{M_t} \quad \text{and}$$

$M_t^E \equiv M_t$), where $B_{t,t+n}$ is the amount of *n-period* bond available in the market and

μ_t is the ordinary money growth rate. It is noticeable that μ_t^E would jump (drop) in a

more volatile way when a yield of a certain bond hits (escapes from) the zero level.

Given that the effect of increased μ_t is negative in the short- run (the liquidity effect) and positive in the long run (the Fisher effect), then a monetary system itself has a automatic mechanism of returning to a positive interest rate as follows: Once a type of bond hits zero, then the total nominal value of the bond issue is added to the effective money stock, which in turn gives downward pressure on the interest rates of bonds with near maturities. Such a tendency of the yield curve approaching the zero line would continue until the short-run negative liquidity effect coming from new entrants to the category of the effective money stock (M_1^E) dominates the long-run Fisher effect arising from the accumulation of M_1^E . So far we have assumed that the monetary authority keeps the money growth rate μ_t constant. Considering that the monetary authority is able to speed up the money growth rate μ_t , then the time required to return to the positive yield curve will be shorter.

V. Concluding Remarks

Our paper explores a transmission mechanism of monetary policy through bond market. Based on the assumption of delayed responses of economic agents to monetary shocks, we

derive a system of equations relating the term structure of interest rates with the past history of money growth. The equations are empirically tested with the US data after some modifications. Impulse response functions of various yield rates with respect to monetary shocks as well as to the short-term interest rate (such as federal fund rate in the US) reveal that the reactions of the yield rates may vary across the bonds with different maturities in terms of directions as well as in terms of magnitudes. From this observation, we find that the policy of maintaining a certain level of the short-term interest rate may lead to different economic consequences depending on the differences of the past monetary policy. Such path-dependency of monetary policy induces that monetary policy targeting a certain shape of the term structure of interest rates could be implemented with a certain time lags.

So far the effects from other omitted exogenous variables are neglected. For example, seemingly significant parameters or variables, such as the variability of money velocity and a shift in consumers' preference, are not fully considered. Such an omission problem would not cause a significant trouble in practice only if omitted variables are deterministic. On the contrary, when the omitted variables are stochastic and are not observable, an algorithm for implementing an optimal monetary policy becomes a usual Kalman filtering setup. This issue is very critical at the stage of application and is expected to be dealt in the following

works.

A. An m-Period Extension of Alvarez, Lucas, and Weber (2001)

Our model is an adapted version of Alvarez, Lucas and Weber (2001). Consider an economy where exist two types of agents-bond-market participant and non-participant.

Regardless of the type, both group have the same intertemporal utility function.

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t U(C_t), \quad \text{where} \quad U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$$

λ portion of the population participates in bond trading and the $(1-\lambda)$ portion does not.

The aggregate production of this economy is y_t .

$$y_t = \lambda C_t^T + (1 - \lambda) C_t^N + \frac{T_t}{P_t},$$

where C_t^T and C_t^N are consumption of the trader and the non-trader each and T_t

is the nominal value for lump-sum tax payment. The budget constraint for the non-trader is

$$P_t C_t^N = \sum_{j=0}^m v_{t-j,t} P_{t-j} y_t, \quad \text{where} \quad \sum_{j=0}^m v_{t,t+j} = 1.$$

At each period he sells his product in the market and receives cash in return ($P_t y$). He allocates this proceeds across $m+1$ periods on consumption with the proportion of

$v_{t,t+j}$, $j = 0, 1, \dots, m$. Another more realistic interpretation of this m-period-ahead CIA feature is that $v_{t,t+j}$, $j = 0, 1, \dots, m$ is the proportion of consumers who need j period time lag in responding to monetary shocks.

On the other hand, the trader spends his money not only on consumption but also on bond trading.

$$\begin{aligned}
 P_t C_t^T &= \sum_{j=0}^m v_{t-j,t} P_{t-j} y_t + \frac{1}{\lambda} (B_t - (\frac{1}{1+r_t}) B_{t+1} - T_t) \\
 &= \sum_{j=0}^m v_{t-j,t} P_{t-j} y_t + \frac{M_t - M_{t-1}}{\lambda}
 \end{aligned}$$

Bond and money supplies satisfy

$$B_t - (\frac{1}{1+r_t}) B_{t+1} - T_t = M_t - M_{t-1},$$

where the government levies the lump-sum tax τ_t on the trader only. The effect of money stock increment would be used either in purchasing bonds or in reducing tax burden.

The goods market equilibrium is attained when the next equation holds:

$$P_t C_t = (1 - \lambda) P_t C_t^N + \lambda P_t C_t^T = P_t y_t$$

Combining the above equations, we obtain

$$\begin{aligned}
 P_t y_t &= \sum_{j=0}^m v_{t-j,t} (P_{t-j} y_{t-j}) + M_t - M_{t-1} \\
 &= M_{t-1} + v_{t,t} P_t y_t + M_t - M_{t-1} \\
 &= V_{t,t} P_t y_t + M_t
 \end{aligned}$$

Accordingly, the equation of exchange is written as

$$M_t \frac{1}{1-v_{t,t}} = P_t y_t .$$

Thus, $v_{t,t}$ can be understood as the money velocity.

From the above equations, we represent the consumption of the trader in the function of money growth rates. Here it is noteworthy that we are interested in the consumption of the trader because in the bond market only the marginal utility of the trader matters for the determination of a yield curve.

$$C_t^T = \frac{1}{\lambda} y_t - \frac{(1-\lambda)}{\lambda} C_t^N = \frac{1}{\lambda} y_t - \frac{(1-\lambda)}{\lambda P_t} \sum_{j=0}^m v_{t-j,t} P_{t-j} y_{t-j}$$

where $\mu^t = (\mu_t, \dots, \mu_{t-m})$, and $v^t = (v_{t,t}, \dots, v_{t-m,t})$. Then, the equilibrium nominal interest rate must satisfy the following marginal condition

$$\left[\frac{1}{1+r_{t,t+k}} \right]^k = E_t \left[\frac{U'(C_{t+k}^T) \frac{(1-v_{t,t})}{(1-v_{t+k,t+k})}}{U'(C_t^T) \prod_{i=0}^k (1+\mu_{t+i})} \right]$$

Notably, the consumption plugged in the above equation is the consumption of trader's, neither that of non-trader's nor the aggregate consumption. This is a way of inducing distributional effect between the trader and the non-trader groups, which in turn leads to the

short-term liquidity effect.

For simplicity, we assume $y_t = y$ and $\tau_t = 0$ for all t . Then,

$$\begin{aligned}
C_t^T &= \frac{1}{\lambda} y \left\{ 1 - (1-\lambda) \sum_{j=0}^m v_{t-j,t} \frac{P_{t-j}}{P_t} \right\} \\
&= \frac{1}{\lambda} y \left\{ 1 - (1-\lambda) \sum_{j=1}^{m-1} v_{t-j+1,t} \frac{1}{\prod_{i=1}^j (1+\mu_{t-i+1})} \frac{(1-v_{t,t})}{(1-v_{t-j,t-j})} \right\} \\
&\equiv c(\mu_t, \dots, \mu_{t-m}, v_{t,t}, \dots, v_{t-m,t}) y \equiv c(\mu^t, v^t) y, \\
\left[\frac{\frac{1}{1+r_{t,t+k}}}{\frac{1}{1+\rho}} \right]^k &= E_t \left[\frac{U'(c(\mu^{t+k}, v^{t+k}) y)}{U'(c(\mu^t, v^t) y)} \frac{1}{\prod_{i=0}^k (1+\mu_{t+i})} \frac{(1-v_{t,t})}{(1-v_{t+k,t+k})} \right] \quad (\text{A-1}) \\
&= E_t \left[\left(\frac{c(\mu^{t+k}, v^{t+k}) y}{c(\mu^t, v^t) y} \right)^{-\gamma} \frac{1}{\prod_{i=0}^k (1+\mu_{t+i})} \frac{(1-v_{t,t})}{(1-v_{t+k,t+k})} \right]
\end{aligned}$$

We assume that the velocity of money ($v_{t,t}$) is constant or exogenously given and the money increase is directed towards the purchase of bonds in the financial market. On the other hand, the last line of (A-1) enables us to briefly analyze the effect of a change in $v_{t,t}$ on the term structure of interest rates.

Consider the liquidity trap as an extreme case, in which any interest rates would not be affected by an increase in money stock. This phenomenon can arise in the economy of

(A-1) exactly when the increase of μ_t is cancelled out by the decrease of $v_{t,t}$. Under the situation like this, the only policy option the government can take is to increase expenditure by speeding up the money growth rate. Then, the market interest rates would go higher following the money increase. It is notable that such a way of monetary expansion transmits a stimulus not through the bond market but through the goods market. The shift of the term structure of interest rates following the monetary expansion is attributed to a new equilibrium in the goods market, which works in an opposite direction to the usual propagation mechanism of open market operation. Anyway, this suggests a way of escaping from the liquidity trap with monetary policy²⁹

Taking the first order approximation of $\log c(\mu^t, v^t)$ around the point $(\underline{0}, \underline{v})$, we obtain

$$\log c(\mu^t, v^t) \cong \left(\frac{1-\lambda}{\lambda} \right) \left\{ \sum_{j=0}^m \bar{v} \left(\sum_{i=0}^j \mu_{t-i} \right) + f(v^t) \right\}$$

$$U'(c(\mu^t, v^t)y) = \exp \left[-\gamma \left(\frac{1-\lambda}{\lambda} \right) \left\{ \sum_{j=0}^m \bar{v} \left(\sum_{i=0}^j \mu_{t-i} \right) + f(v^t) \right\} \right] y^{-\gamma} \quad (\text{A-2})$$

Substituting (A-2) into (A-1) and taking log by both sides, then we obtain

²⁹ Though the arguments in this paragraph consider neither Ricardian equivalence nor crowding-out effect explicitly, the

$$\begin{bmatrix} r_{t,t+1} \\ r_{t,t+2} \\ \dots \\ r_{t,t+n-1} \\ r_{t,t+n} \end{bmatrix} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} & \dots & \phi_{1,m-1} & \phi_{1,m} \\ \phi_{2,1} & \phi_{2,2} & \dots & \dots & \dots & \phi_{2,m} \\ \phi_{3,1} & \dots & \dots & \phi_{i,j} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \phi_{n-1,m} \\ \phi_{n,1} & \phi_{n,2} & \dots & \dots & \phi_{n,m-1} & \phi_{n,m} \end{bmatrix} \begin{bmatrix} \mu_t \\ \mu_{t-1} \\ \dots \\ \mu_{t-m+2} \\ \mu_{t-m+1} \end{bmatrix} + R(v^t) \quad (\text{A-3})$$

or simply

$$\Gamma_t = \Phi_t \Delta_t + R(v^t),$$

where R_t is a $n \times 1$ vector, and Δ_t a $m \times 1$ vector, and Φ_t a $n \times m$ matrix.

The coefficients of the matrix in (A-3) are derived from (A-1) and (A-2). For

$$1 \leq j < m - i + 1,$$

$$\phi_{i,j} = -\gamma \left(\frac{1-\lambda}{\lambda} \right) v^-$$

For $m \geq j \geq m - i + 1 \geq 1$,

$$\phi_{i,j} = \gamma \left(\frac{1-\lambda}{\lambda} \right) v^- \left\{ \frac{m-j+1}{i} \right\} > 0$$

Neglecting that the expectation for the future monetary policies does not change, then the coefficients of Φ indicates that cross-sectionally an increase in μ_t lowers the yield rates of bonds with shorter maturities than $m+1$ periods while the yield rates of the bonds with maturities longer than m are raised. Combining these two, we can deduce that there is a slope change in the yield curve between m and $m+1$. Accordingly, the liquidity effect

equations from our model can test their validity.

view is supported for bonds with maturities shorter than m and the Fisher's view is valid for bonds with maturities longer than $m+1$. In addition, the cross-time effect of μ_t changes signs from negative to positive, which also confirms that in the long run Fisher effect prevails.

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Appendix B. Tables

[Table 1] Regression^{B1} results of Eq (6) (monthly)

	d_fedfundr	d_tb3mon	d_tb6mon	D_tb1yr	d_longbd
slope	-127.9254 ** (27.96452) ^{B2}	-94.39297 ** (20.4626)	-85.78889 ** (18.60477)	-73.42162 ** (15.1847)	-30.08043 ** (8.06099)
curv	110.9467 ** (24.56312)	63.6107 ** (19.44036)	53.95926 ** (17.83941)	44.23584 ** (15.38967)	12.22965 (9.06536)
third	-29.04991 ** (7.56772)	-9.744747 (6.55566)	-7.625036 (6.007119)	-5.851174 (5.438865)	-0.8182089 (3.474307)
R-square	0.1701	0.1994	0.1947	0.1795	0.1091

** P-value < 0.01

* P-value < 0.05

[Table 2] Cross-sectional variations in yield rates in response to 1% increase in money growth rate (monthly)

	fedfundr	tb3mon	tb6mon	tb1yr	ltgovtbd
Estimates					
0	-46.0286	-40.527	-39.4547	-35.037	-18.669
1	-6.81832	-3.59419	0.745467	2.503457	8.075756
2	23.79699	34.37646	31.08415	26.68232	9.775025
Lower (95%)					
0	-67.9137	-56.9441	-54.4702	-47.6539	-25.8992
1	-24.7057	-19.6211	-12.7498	-9.68692	0.594773
2	-2.21603	13.99041	13.67072	12.30465	2.59023
Upper(95%)					
0	-24.1434	-24.1099	-24.4391	-22.42	-11.4388
1	11.06904	12.43267	14.24075	14.69383	15.55674
2	49.81001	54.7625	48.49759	41.05999	16.95982

^{B1} We run regression equation by equation with Newey-West estimates of variances. The same method is applied in the estimation of Eq (7), the output from which is summarized in [Table 3].

^{B2} All the numbers in parentheses are estimates of standard errors

[Table 3] Regression results of Eq (7) (quarterly)

	fedfundr	tb3mon	Tb6mon	tb1yr	ltgovtbd
m1sl_g	-76.99609 *	-63.07492 *	-64.10236 **	-62.75473 **	-17.39448
	(31.5835)	(25.66712)	(23.21046)	(19.82346)	(13.17486)
slope	-128.2924	-100.8881	-101.4461 *	-95.13843 *	-99.38094 **
	(70.01885)	(56.9168)	(51.87965)	(44.61863)	(30.2464)
curv	238.1501 **	176.1443 **	183.5877 **	183.3664 **	171.3927 **
	(60.87203)	(51.02896)	(48.52384)	(42.76306)	(29.70285)
third	-174.7073 *	-127.8155 *	-135.675 **	-139.2755 **	-130.5185 **
	(68.19693)	(56.05653)	(50.81901)	(43.75916)	(28.82497)
fourth	51.36435	36.92932	39.26172 *	40.61439 *	37.68054 **
	(31.71773)	(25.7025)	(22.95705)	(19.4546)	(12.26265)
M1sl	-0.03379 **	-0.028509 **	-0.028257 **	-0.026989 **	-0.021644 **
	(0.0026121)	(0 .0021817)	(0.0020829)	(0.001892)	(0.0013401)
gdpdef	0.325513 **	0.2956538 **	0.2923121 **	0.2854734 **	0.3406851 **
	(0.0372945)	(0 .0295202)	(0.0275346)	(0 .0245559)	(0.0179704)
rgdp	0.001349 **	0.0008278 *	0.0008153 *	0.0006974 *	-0.000806 **
	(0.0004773)	(0.0003734)	(0.0003648)	(0.000346)	(0 .0002619)
rgdp_g	-78.2219 **	-47.51647 **	-45.50964 **	-38.88831 **	-18.04747 *
	(18.48785)	(13.47141)	(12.74745)	(11.56509)	(8.427331)
_cons	-0.615336	0.1695507	0.4198887	0.7155139	2.70567 **
	(1.053493)	(0.8676736)	(0 .8527895)	(0.8149884)	(0 .6244242)
R-square	0.6753	0.6953	0.7083	0.7264	0.8399

[Table 4] Cross-sectional variations* in yield rates in response to 1% increase in money
growth rate (quarterly)

	Fedfundr	tb3mon	tb6mon	tb1yr	ltgovtbd
Estimates					
0	-90.4813	-78.7049	-78.3741	-73.1878	-38.22073
1	-29.3433	-15.6714	-15.7512	-16.2255	-2.571018
2	22.21436	14.27385	12.13303	9.226298	5.920387
3	-30.7501	-19.9018	-21.3719	-23.1821	-20.20366
Lower (95%)					
0	-129.246	-110.433	-107.024	-97.8495	-56.89042
1	-75.2743	-53.2482	-50.7906	-47.0053	-23.88314
2	-35.2496	-31.3383	-30.6236	-27.8615	-18.46947
3	-166.433	-128.475	-118.122	-104.288	-69.49829
Uppper(95%)					
0	-51.7168	-46.9771	-49.7239	-48.5261	-19.55103
1	16.58761	21.90539	19.28824	14.55418	18.7411
2	79.67828	59.88599	54.8897	46.31408	30.31025
3	104.9329	88.67106	75.37808	57.92365	29.09098

* The effect from money stock (m1sl) is not included.

[Table 5] Regression result of Equation (3)

(Quarterly, 163 observations)

	real GDP growth rate (t+1)
fedfundr (t)	-0.0058828 ** (0.001273)
tb3mon (t)	0.0177324* (0.0060302)
tb6mon (t)	-0.0250236 * (0.0120913)
tb1yr (t)	0.01445 (0.00737)
ltgovtbd (t)	-0.0007248 (0.0007363)
constant	0.0106001 ** (0.0021766)
R-square	0.2610

** P-value < 0.01

* P-value < 0.05

[Table 6] Covariances of different ordered moments of money growth

(Monthly, 487 observations)

	m1sl_g	Slope	curv	Fourth	fifth
m1sl_g	0.000025				
slope	0.000011	0.000022			
curv	6.3e-06	0.000028	0.000057		
fourth	1.3e-06	0.00003	0.000087	0.000173	
fifth	-6.2e-06	0.000031	0.000089	0.00018	-0.000212

(Quarterly, 159 observations)

	m1sl_g	Slope	curv	third	Fourth
m1sl_g	0.00012				
slope	0.000039	0.000076			
curv	0.000025	0.000101	0.000202		
third	0.000018	0.000119	0.000322	0.000645	
fourth	0.000068	0.000123	0.000461	0.001081	0.002076

[Table 7] Results from running simultaneously a system of equations in Eq (7)
with a Taylor type federal fund rate determination rule

(Quarterly, 159 observations)						
	fedfunds	tb3mon	Tb6mon	tb1yr	ltgovtbd	Fedfunds
m1sl_g	-45.3135 *	-39.0019 *	-42.0153 **	-43.7244 **	-8.28862	
	(20.7154)	(16.4791)	(15.95189)	(14.77506)	(11.58109)	
slope	-119.533 **	-94.2325 **	-95.3396 **	-89.8770 **	-96.8634 **	
	(46.1740)	(36.7196)	(35.5328)	(32.89808)	(25.74425)	
Curv	193.915 **	142.5335 **	152.7499 **	156.7963 **	158.6791 **	
	(49.3225)	(39.2193)	(37.94754)	(35.12915)	(27.47552)	
third	-135.256 **	-97.8397 **	-108.172 **	-115.579 **	-119.180 **	
	(44.7487)	(35.5918)	(34.44723)	(31.89939)	(24.98296)	
fourth	38.5775 *	27.2136 *	30.34752 *	32.93385 **	34.00546 **	
	(16.8957)	(13.43996)	(13.0092)	(12.0487)	(9.44168)	
m1sl	-0.02637 **	-0.02287 **	-0.02309 **	-0.02254 **	-0.01951 **	
	(0.001948)	(0.0015466)	(0.0014939)	(0.0013801)	(0.0010704)	
gdpdef	0.239849 **	0.230565 **	0.232593**	0.234019 **	0.316065 **	
	(0.029514)	(0.0234024)	(0.022576)	(0.0208244)	(0.0160479)	
rgdp	0.001205 **	0.0007184 *	0.0007149 *	0.0006109 *	-0.00085 **	
	(0.000413)	(0.000327)	(0.0003151)	(0.0002903)	(0.0002225)	
rgdp_g	-72.903 **	-43.47517 **	-41.8018 **	-35.6936 **	-16.51881	
	(15.3313)	(12.18547)	(11.78481)	(10.90345)	(8.508546)	
_cons	0.957174	1.364377	1.516141 *	1.660053 **	3.157624 **	4.046584 **
	(0.882548)	(0.7004436)	(0.6763709)	(0.6246324)	(0.4837461)	(0.3323494)
gdpdef_g						253.7386 **
						(27.99782)
lgdp_gap						-18.9074 **
						(6.682002)
R-square	0.6413	0.6657	0.6823	0.7049	0.8347	0.4410

** P-value < 0.01

* P-value < 0.05

[Table 8] Autoregressive movements of money growth rate induced by a Taylor type short-term interest rate policy function

(1) Lag length selection order criteria (quarterly, 159 observations)

	Lag 0	Lag 1	Lag2	Lag3	Lag4
LL	510.685	551.373	555.103	557.026	557.036
LR		81.376	7.459	3.846 *	0.019
FPE	0.0001038	0.000063	0.0000609	0.0000602 *	0.0000609
AIC	-6.33566	-6.83488	-6.86922	-6.88083 *	-6.86837
HQIC	-6.28079	-6.77218	-6.79868	-6.80245 *	-6.78215
SBIC	-6.20055	-6.68047	-6.69551 *	-6.68782	-6.65606

(2) Estimation results

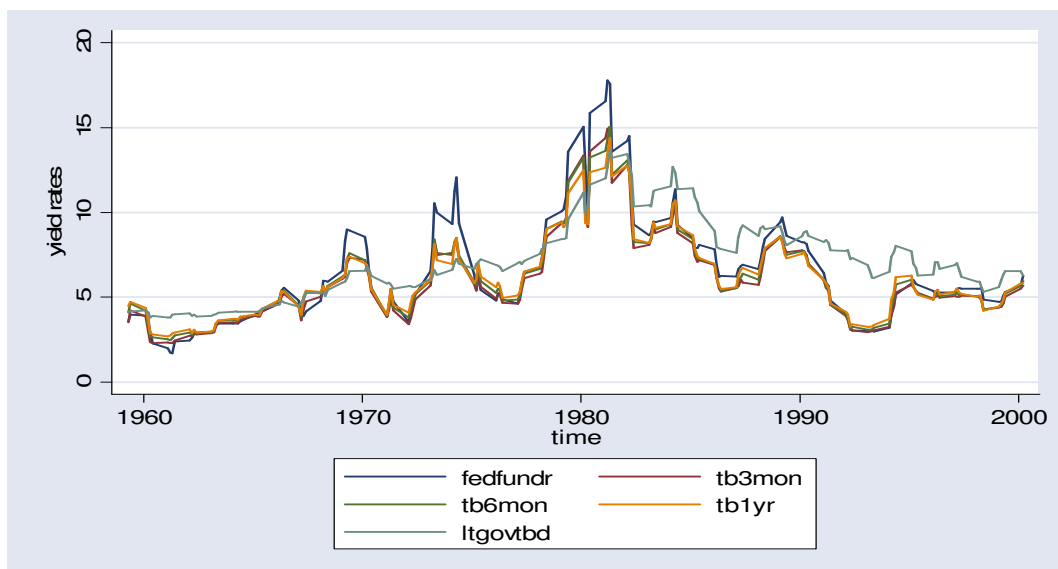
	Coefficient	Std. Err	Z
L1	0.4928921	0.0751039	6.56 **
L2	0.2061028	0.0738131	2.79 **
Lgdp_gap	-0.0341836	0.0347653	-0.98
Gdpdef_g	-0.281095	0.1318882	-2.13 *
rgdp_g	-0.1517057	0.0712509	-2.13 *
gdpdef	0.0002253	0.000173	1.30
Rgdp	2.17e-06	2.57e-06	0.84
M1sl	-0.0000343	9.78e-06	-3.51 **
_cons	0.0014172	0.0041283	0.34

** P-value < 0.01

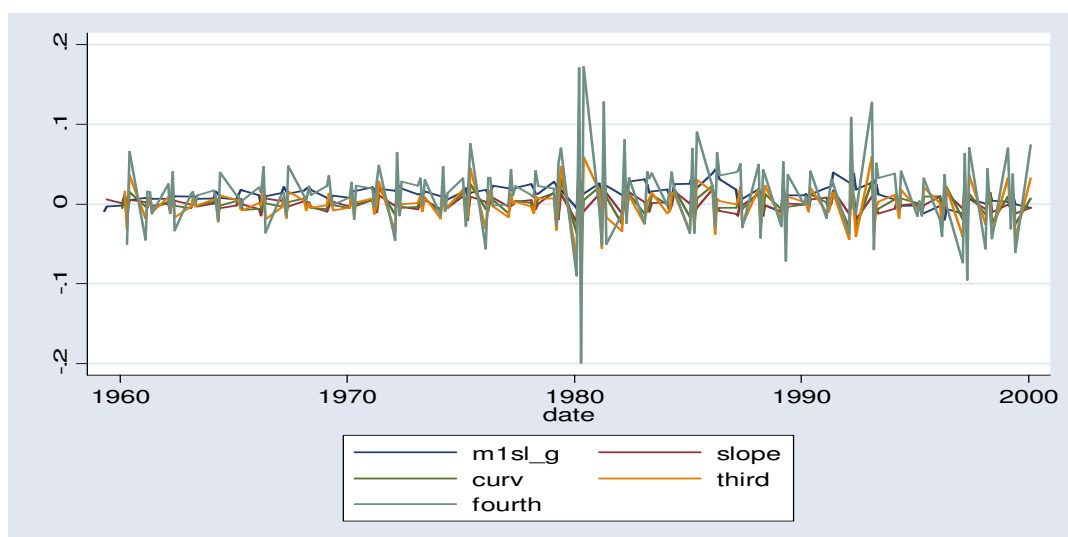
* P-value < 0.05

Appendix C. Figures

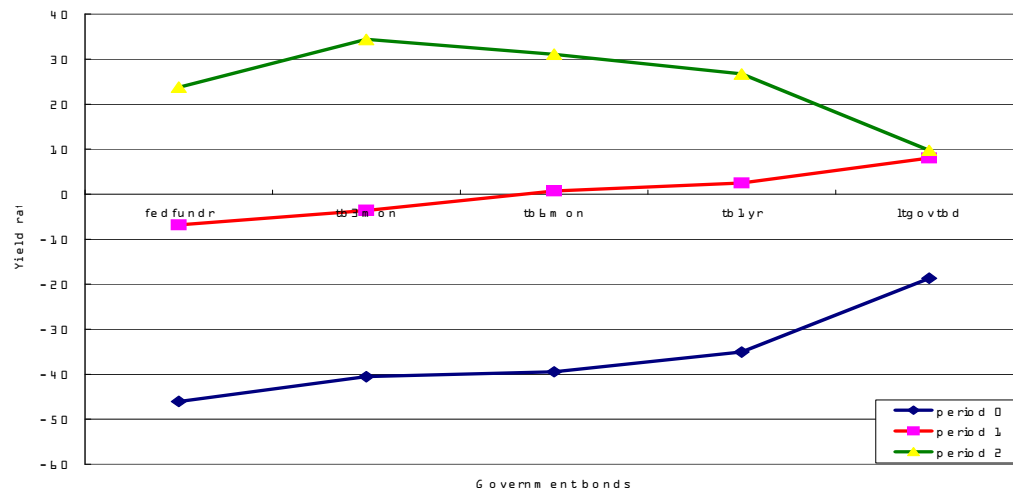
[Figure1] The movements of yield rates in the US during 1960-2000 (quarterly)



[Figure 2] The movements in the higher order moments of M1 growth rate (quarterly)



[Figure 3] Cross-sectional variations in the term structure of interest rates in response to 1% increase in the money growth rate (monthly)



[Figure 4] Cross-sectional variations in the term structure of interest rates in response to 1% increase in the money growth rate (quarterly)

